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Smooth bifurcation branches of solutions for a Signorini problem

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ABSTRACT

We study a bifurcation problem for the equation $\Delta u + \lambda u + g(\lambda, u)u = 0$ on a rectangle with Signorini boundary conditions on a part of one edge and mixed (zero Dirichlet and Neumann) boundary conditions on the rest of the boundary. Here $\lambda \in \mathbb{R}$ is the bifurcation parameter, and g is a small perturbation. We prove, under certain assumptions concerning an eigenfunction u_0 corresponding to an eigenvalue λ_0 of the linearized equation with the same nonlinear boundary conditions, the existence of a local smooth branch of nontrivial solutions bifurcating from the trivial solutions at λ_0 in the direction of u_0. The contact sets of these nontrivial solutions are intervals which change smoothly along the branch. The main tool of the proof is a local equivalence of the unilateral BVP to a system consisting of a corresponding classical BVP and of two scalar equations. To this system classical Crandall–Rabinowitz type local bifurcation techniques (scaling and Implicit Function Theorem) are applied.

1. Introduction

Let $\Omega := (0, 1) \times (0, \ell)$ be a rectangle with $\ell > 0$. Let its boundary be divided into a "Dirichlet part" $\Gamma_D := (\{0\} \times (0, \ell)) \cup (\{1\} \times (0, \ell))$, a "unilateral part" $\Gamma_U := ((\gamma_1, \gamma_2) \times \{0\}) \subset ((0, 1) \times \{0\})$ with $0 < \gamma_1 < \gamma_2 < 1$ and a "Neumann part" $\Gamma_N := \partial\Omega \setminus \left(\overline{\Gamma_D} \cup \overline{\Gamma_U}\right)$ (see Fig. 1). We will study bifurcation from the trivial solutions of the Signorini boundary value problem

$$\Delta u + \lambda u + g(\lambda, u)u = 0 \quad \text{in } \Omega, \tag{1.1}$$

$$u = 0 \quad \text{on } \Gamma_D, \qquad \partial_\nu u = 0 \quad \text{on } \Gamma_N, \tag{1.2}$$

$$u \le 0, \qquad \partial_\nu u \le 0, \qquad u\partial_\nu u = 0 \quad \text{on } \Gamma_U, \tag{1.3}$$

where λ is a real bifurcation parameter and $g : \mathbb{R}^2 \to \mathbb{R}$ is a C^1-smooth function,

$$g(\lambda, 0) = 0 \quad \text{for all } \lambda \in \mathbb{R}, \tag{1.4}$$

$$|\partial_u g(\lambda, u)| + |\partial_\lambda g(\lambda, u)| \le C(1 + |u|^q) \quad \text{for all } (\lambda, u) \in \mathbb{R}^2 \tag{1.5}$$

with some $C > 0$ and $q > 1$. Furthermore, we will assume that we are given an eigenvalue $\lambda_0 > 0$ and a corresponding normalized eigenfunction u_0 for the (nonlinear) eigenvalue problem

$$\Delta u + \lambda u = 0 \quad \text{in } \Omega \tag{1.6}$$

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