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Semilinear parabolic problems in thin domains with a highly oscillatory boundary

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ABSTRACT

In this paper, we study the behavior of the solutions of nonlinear parabolic problems posed in a domain that degenerates into a line segment (thin domain) which has an oscillating boundary. We combine methods from linear homogenization theory for reticulated structures and from the theory on nonlinear dynamics of dissipative systems to obtain the limit problem for the elliptic and parabolic problems and analyze the convergence properties of the solutions and attractors of the evolutionary equations.

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1. Introduction

In this paper, we are interested in analyzing the asymptotic behavior of solutions of parabolic PDEs in a thin domain with a highly oscillatory behavior in its boundary, as depicted in Fig. 1.

To state the problem, let $g : \mathbb{R} \mapsto \mathbb{R}$ be a $\mathcal{C}^1$, L-periodic positive function with $0 < g_0 \le g(x) \le g_1$ for all $x \in \mathbb{R}$ where $g_1 = \max_{x\in\mathbb{R}}\{g(x)\}$ and consider the following bounded open set

$$R^\epsilon = \{(x, y) \in \mathbb{R}^2 \mid x \in (0, 1) \text{ and } 0 < y < \epsilon g(x/\epsilon)\} \tag{1.1}$$

where $\epsilon > 0$ is arbitrary. For sake of notation, let us denote the *oscillatory* part of the boundary by $\partial_o R^\varepsilon = \{(x, \varepsilon g(x/\varepsilon)) : 0 < x < 1\}$ the *fixed* part of the boundary by $\partial_f R^\varepsilon = \{(x, 0) : 0 < x < 1\}$ and the *lateral* part of the boundary as $\partial_l R^\varepsilon = \{(0, y) : 0 < y < \varepsilon g(0)\} \cup \{(1, y) : 0 < y < \varepsilon g(1/\varepsilon)\}$. If we need to distinguish between the two parts of the lateral boundary (left and right), we will write $\partial_{ll} R^\varepsilon$ and $\partial_{lr} R^\varepsilon$.

In the thin domain R^ϵ we consider the following semilinear parabolic evolution equation

$$\begin{cases} w_t^\epsilon - \Delta w^\epsilon + w^\epsilon = f(w^\epsilon) & \text{in } R^\epsilon,\ t > 0 \\ \dfrac{\partial w^\epsilon}{\partial \nu^\epsilon} = 0 \quad \text{on } \partial R^\epsilon \end{cases} \tag{1.2}$$

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