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Fractal Haar system

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ABSTRACT

The Haar system is an alternative to the classical Fourier bases, being particularly useful for the approximation of discontinuities. The article tackles the construction of a set of fractal functions close to the Haar set. The new system holds the property of constitution of bases of the Lebesgue spaces of p-integrable functions on compact intervals. Likewise, the associated fractal series of a continuous function is uniformly convergent. The case $p = 2$ owns some peculiarities and is studied separately.

1. Introduction

One of the best contributions of the fractal geometry and the chaos theory to science is the proposition of the first and systematic quantitative approximation in the study of the irregularity (in the spatial and temporal senses) in both contexts, inside the scope of pure mathematics and the real world, including the analysis of subjects like turbulence, materials, bioelectric recordings, acoustic pollution, etc.

Concerning the field of economy, many analysts think that the behavior of the stock market can be modeled by means of dynamical systems and fractal sets. For instance, Mandelbrot noted that the financial series display an interdependence extended over long periods of time and they are not a mere background noise [1]. The intra-day data seem somewhat random but it is clear that the long term prices follow a well defined trend.

The appearance and development of the fractal functions make the classification between deterministic and random processes increasingly vague. Mandelbrot proposed a model initially studied by Weierstrass [1] that simulates diverse stochastic motions. Depending on the selection of the parameters, one obtains a Gaussian random function, a Brownian fractal or a model for $1/f$ noise [2–4]. In this instance, the irregularity of the signal is correlated with the fractal dimension but uncorrelated to the randomness of the phase coefficients.

Our humble contribution to the theory has been the definition of "rough" functions, defined as perturbation of the classical (polynomial, trigonometric, etc.) functions. This deformation is performed by means of an iterated function system [5–7]. These maps tend to bridge the gap between the smoothness of the classical mathematical objects and the pseudo-randomness of the experimental data, breaking in this way their apparent contradiction.

Nowadays the computation of fractal exponents and dimensions is used in the most diverse fields of science and technology (see for instance [8–12] for cardiosignals). In the case of electrocardiographic recordings, some indices have acquired a pre-diagnostic and predictive importance. The reduction of complexity (measured by means of fractal dimensions) with the disease is found in very different contexts.

On the other hand, it is unnecessary to stress the transcendence of the wavelet theory in signal processing and, in particular, in the analysis of non-stationary data (see for instance [13]). The theory has exceeded the scope of mathematics

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