



Classification and evolution of bifurcation curves for a multiparameter p -Laplacian Dirichlet problem[☆]

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ARTICLE INFO

Article history:

Received 5 October 2010

Accepted 28 February 2011

Accepting Editor: Ravi Agarwal

MSC:

34B18

74G35

Keywords:

Evolution

Bifurcation curve

Exact multiplicity

Positive solution

p -Laplacian

Time map

ABSTRACT

We study the classification and evolution of bifurcation curves for the multiparameter p -Laplacian Dirichlet problem

$$\begin{cases} (\varphi_p(u'(x)))' + \lambda u^q \left(\sum_{k=1}^n a_k u^{r_k} \right)^{-1} = 0, & -1 < x < 1, \\ q > 0, & 0 = r_1 < r_2 < \dots < r_n, n \geq 2, \quad a_k > 0 \text{ for } k = 1, 2, \dots, n, \\ u(-1) = u(1) = 0, \end{cases}$$

where $p > 1$, $\varphi_p(y) = |y|^{p-2}y$, $(\varphi_p(u'))'$ is the one-dimensional p -Laplacian, and $\lambda > 0$ is a bifurcation parameter, and $q > 0$ is an evolution parameter. We give a classification of totally five qualitatively different bifurcation curves for different $q > 0$. More precisely, we prove that, on the $(\lambda, \|u\|_\infty)$ -plane, each bifurcation curve is either a monotone curve if $q \in (0, p - 1] \cup [r_n + p - 1, \infty)$ or has exactly one turning point where the curve turns to the right if $q \in (p - 1, r_n + p - 1)$. Hence the problem has at most two positive solutions for each $\lambda > 0$. We also show evolution of five bifurcation curves as q varies from 0^+ to ∞ .

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1. Introduction and main result

In this paper we mainly study the classification and evolution of bifurcation curves of positive solutions for the multiparameter p -Laplacian Dirichlet problem

$$\begin{cases} (\varphi_p(u'(x)))' + \lambda u^q \left(\sum_{k=1}^n a_k u^{r_k} \right)^{-1} = 0, & -1 < x < 1, \\ q > 0, & 0 = r_1 < r_2 < \dots < r_n, \quad n \geq 2, \quad a_k > 0 \text{ for } k = 1, 2, \dots, n, \\ u(-1) = u(1) = 0, \end{cases} \quad (1.1)$$

where $p > 1$, $\varphi_p(y) = |y|^{p-2}y$, $(\varphi_p(u'))'$ is the one-dimensional p -Laplacian, nonnegative constants r_1, r_2, \dots, r_n , and a_1, a_2, \dots, a_n with $n \geq 2$ are given, $\lambda > 0$ is a bifurcation parameter, and $q > 0$ is an evolution parameter.

[☆] This work is partially supported by the National Science Council of the Republic of China under grant No. 95-2115-M-007-013-MY3.

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