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Radial solutions of Dirichlet problems with concave–convex nonlinearities

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ABSTRACT

We prove the existence of a double infinite sequence of radial solutions for a Dirichlet concave–convex problem associated with an elliptic equation in a ball of $\mathbb{R}^n$. We are interested in relaxing the classical positivity condition on the weights, by allowing the weights to vanish. The idea is to develop a topological method and to use the concept of rotation number. The solutions are characterized by their nodal properties.

1. Introduction

In this paper, we are concerned with the Dirichlet problem

$$\begin{cases} \Delta u(x) + q(|x|)|u(x)|^{\delta-1}u(x) + p(|x|)\,|u(x)|^{\gamma-1}u(x) = 0 & x \in \Omega \\ u(x) = 0 & x \in \partial\Omega, \end{cases} \tag{1.1}$$

where Ω is the unit ball in $\mathbb{R}^N$ with $N \geq 3$ and $p, q : [0, 1] \to \mathbb{R}$ are C^1 functions. Moreover, we assume that $0 < \gamma < 1 < \delta$; hence, the nonlinearity we are dealing with is of concave–convex type.

We are interested in the search for solutions of (1.1) with prescribed nodal properties. This kind of problem has been intensively studied in the literature; for a quite exhaustive bibliography, we refer readers to [1–4] and the references therein.

The nonlinearity we are studying combines two different aspects; on the one hand it is superlinear at infinity, and on the other hand it is sublinear near zero. As a consequence, the problem inherits the qualitative features of both superlinear and sublinear problems. Starting from the pioneering paper [5], many authors have studied this problem, under various assumptions on the coefficients q and p. The main question is to prove the existence of four sequences of nodal solutions of (1.1), two of them with large norm and two with small norm.

This has been proved, for instance, in [2] and [6, Theorem 1.5] when q and p are strictly positive. In the one-dimensional setting, we wish to mention, among others, the contribution of [7,8] and [9, Theorem 4], providing a detailed study of the exact structure of the solutions in the presence of positive constant weights. [7] also treats the p-strictly-negative case, showing that, in this context, no more than two sequences of solutions can be achieved. More generally, the positiveness of each weight on a subset of Ω of nonzero measure is a necessary condition to guarantee the existence of four sequences of solutions. It is still an open question whether this condition is also sufficient.

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