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Lipschitzian stability of parametric variational inequalities over generalized polyhedra in Banach spaces

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ABSTRACT

This paper concerns the study of solution maps to parameterized variational inequalities over generalized polyhedra in reflexive Banach spaces. It has been recognized that generalized polyhedral sets are significantly different from the usual convex polyhedra in infinite dimensions and play an important role in various applications to optimization, particularly to generalized linear programming. Our main goal is to fully characterize robust Lipschitzian stability of the aforementioned solution maps entirely via their initial data. This is done on the basis of the coderivative criterion in variational analysis via efficient calculations of the coderivative and related objects for the systems under consideration. The case of generalized polyhedra is essentially more involved in comparison with usual convex polyhedral sets and requires developing elaborated techniques and new proofs of variational analysis.

1. Introduction

Parametric variational inequalities are among the most important objects in optimization theory and variational analysis; see, e.g., books [1–6] and the references therein. A breakthrough in their study and applications goes back to the seminal work by Robinson [7,8] who treated them as parametric "generalized equations"

$$0 \in f(p,x) + N(x;\Theta) \quad \text{for all } x \in \Theta, \tag{1.1}$$

where $x \in X$ is the decision variable and $p \in Z$ is the parameter taking values in the corresponding Banach spaces. The "base" mapping $f: Z \times X \to X^*$ in (1.1) takes values in the dual space X^* while the set-valued "field" part $N: X \rightrightarrows X^*$ is the normal cone mapping to a convex set $\Theta \subset X$. By the classical definition of the normal cone in convex analysis with $N(x;\Omega) := \emptyset$ if $x \notin \Theta$, the generalized equation form (1.1) is equivalent to the standard form of variational inequalities: for each $p \in Z$ find $x \in \Theta$ such that

$$\langle f(p,x), x-u\rangle \le 0 \quad \text{whenever } u \in \Theta.$$

It has been well recognized that the generalized equation formalism (1.1) is a convenient model to describe parametric complementarity problems, moving sets of optimal solutions to various optimization and equilibrium problems, KKT systems, and the like; see the references mentioned above and the bibliographies therein.

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