



# The Muskat problem with surface tension and a nonregular initial interface

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## ABSTRACT

In this paper, we consider the two-dimensional Muskat problem with surface tension on a free boundary. The initial shape of the unknown interface has a corner point. We prove that the problem has a unique solution in the weighted Hölder classes locally in time and specify the sufficient conditions for the existence of the “waiting time” phenomenon.

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## 1. Introduction

The Muskat problem describes the evolution of an interface between two immiscible fluids in a porous medium under applied pressure gradients. We consider the Muskat problem, which has the following features. First of all, it is a free boundary problem; next, this is a transmission problem, and we take into account the curvature of the interface; and, finally, the initial interface is nonregular, and in particular, there are angular points on the initial interface.

The two-phase free boundary Hele-Shaw problem (the Muskat problem) without surface tension and with a regular initial interface was studied by Yi [1,2], Ambrose [3] and Siegel et al. [4]. A similar problem with surface tension which is proportional to the curvature of the free boundary was investigated by Hong et al. [5].

The one-phase free boundary problem (the Hele-Shaw problem) with surface tension for general smooth initial domains was considered by Chen [6], Escher and Simonett [7], Bazaliy and Friedman [8], and Prokert [9] by various methods.

The first qualitative analysis of the one-phase Hele-Shaw problem without surface tension and with a corner point on an initial interface was carried out by King et al. [10]. In particular, they found that under some initial conditions the interface conserves the corner point for some time (the phenomenon of the “waiting time”). Vasylyeva [11] has proved the existence (locally in time) of solutions with the “waiting time” property in the weighted Hölder spaces for the one-phase Hele-Shaw problem without surface tension and with angle points on the free boundary. A similar problem with surface tension was investigated by Bazaliy and Friedman [12,13].

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