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Asymptotic stability of viscous contact discontinuity to an inflow problem for compressible Navier–Stokes equations☆

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ABSTRACT

This paper is concerned with an initial-boundary value problem for one-dimensional full compressible Navier–Stokes equations with inflow boundary conditions in the half space $\mathbb{R}_+ = (0, +\infty)$. The asymptotic stability of viscous contact discontinuity is established under the conditions that the initial perturbations and the strength of contact discontinuity are suitably small. Compared with the free-boundary and the initial value problems, the inflow problem is more complicated due to the additional boundary effects and the different structure of viscous contact discontinuity. The proofs are given by the elementary energy method.

1. Introduction

This paper is concerned with an "inflow problem" for one-dimensional compressible viscous heat-conducting flows in the half space $\mathbb{R}_+ = (0, \infty)$, which is governed by the following initial-boundary value problem in Eulerian coordinate $(\tilde{x}, t)$:

$$\begin{cases} \tilde{\rho}_t + (\tilde{\rho}\tilde{u})_{\tilde{x}} = 0, \quad (\tilde{x}, t) \in \mathbb{R}_+ \times \mathbb{R}_+, \\ (\tilde{\rho}\tilde{u})_t + (\tilde{\rho}\tilde{u}^2 + \tilde{p})_{\tilde{x}} = \mu \tilde{u}_{\tilde{x}\tilde{x}}, \\ \left(\tilde{\rho}\left(\tilde{e} + \dfrac{\tilde{u}^2}{2}\right)\right)_t + \left(\tilde{\rho}\tilde{u}\left(\tilde{e} + \dfrac{\tilde{u}^2}{2}\right) + \tilde{p}\tilde{u}\right)_{\tilde{x}} = \kappa \tilde{\theta}_{\tilde{x}\tilde{x}} + (\mu \tilde{u}\tilde{u}_{\tilde{x}})_{\tilde{x}}, \\ (\tilde{\rho}, \tilde{u}, \tilde{\theta})|_{\tilde{x}=0} = (\rho_-, u_b, \theta_-) \quad \text{with } u_b > 0, \\ (\tilde{\rho}, \tilde{u}, \tilde{\theta})|_{t=0} = (\tilde{\rho}_0, \tilde{u}_0, \tilde{\theta}_0)(\tilde{x}) \to (\rho_+, u_b, \theta_+) \quad \text{as } \tilde{x} \to \infty, \end{cases} \tag{1.1}$$

where $\tilde{\rho}$, $\tilde{u}$ and $\tilde{\theta}$ are the density, velocity and absolute temperature, respectively; $\mu > 0$ is the viscosity coefficient and $\kappa > 0$ is the heat-conductivity coefficient. Throughout the paper, it is assumed that $\rho_\pm$, u_b and $\theta_\pm$ are prescribed positive

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