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A new development for the Tikhonov Theorem in nonlinear singular perturbation systems ☆

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ABSTRACT

This paper deals with the exponential stability of nonlinear perturbation systems under a new condition. A novel criterion of exponential stability of nonlinear systems is firstly given in a general form. In this criterion, a new kind of characteristic value is introduced, which makes the exponential stability measurable. Based on this criterion, a new development for the Tikhonov Theorem in nonlinear singular perturbation systems is presented.

1. Introduction

Consider the nonlinear singular perturbation system [1]:

$$\dot{x} = F(x, z), \tag{1a}$$

$$\epsilon\dot{z} = G(x, z) + Q(x)u, \tag{1b}$$

where $x \in \mathbb{R}^n$ and $z \in \mathbb{R}^m$ are slow and fast dynamic state variables respectively, $u \in \mathbb{R}^q$ the system input, F and G both vector functions, Q an $m \times q$ matrix, $\epsilon > 0$ a small parameter that is the ratio of the time scale of the fast dynamics to the time scale of the slow dynamics. Generally, (1a) and (1b) are called slow and fast dynamical subsystems respectively.

System (1) is a typical singular perturbation control system. Many physical systems are of this form, for instance, the modified IEEE type-2 voltage regulator [1,2], the armature-controlled DC motor [3], the flexible joint manipulator [4–8], and so on. The main property of these control systems is that the control input u has direct effect on the fast subsystem (1b) only, but the control objective is the slow subsystem (1a) usually. Therefore, if the Jacobian matrix $\frac{\partial F(x,z)}{\partial z}$ is singular, the linearized system of system (1) is uncontrollable. As a result, traditional linearization techniques are not suitable for these kinds of control systems. However, under certain conditions, system (1) can be approximated by two independent subsystems with different time scales, i.e., the boundary-layer system and the reduced system (see the Tikhonov Theorem, [3, Theorem 11.1]). So if we can divide the control input into two proper parts, they appear in the boundary-layer system and the reduced system individually. Therefore, it is able to make system (1) hold some desired capabilities by designing the boundary-layer system and the reduced system respectively. The composite control is based on this idea and at present, most control designs of singular perturbation system are composite control [1,7].

Since the singular perturbation systems are widely applied, their stability problem attracts lots of attention and the Tikhonov Theorem had ever been a main mean in these research [9–11]. The Tikhonov Theorem implies that whether the

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