Nonlinear Analysis 74 (2011) 2695–2719

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

# Dynamics of a class of ODEs more general than almost periodic☆

P.E. Kloeden [a,*], H.M. Rodrigues [b]

[a] Institut für Mathematik, Johann Wolfgang Goethe Universität, D-60054 Frankfurt am Main, Germany

[b] Instituto de Ciências Matemáticas e de Computação, Universidade de São Paulo, C.P. 668, CEP: 13560-970 São Carlos(SP), Brazil

ARTICLE INFO

*Article history:*
Received 4 September 2008
Accepted 23 December 2010

Dedicated to the memory of Jack K. Hale

*MSC:*
Primary 34C25
34C27
34C30
Secondary 34G20
34E10

*Keywords:*
Periodic solution
Almost periodic solution
Almost automorphic solution
Sequential set
Stable manifold
Unstable manifold
Pullback attractor

ABSTRACT

A temporally global solution, if it exists, of a nonautonomous ordinary differential equation need not be periodic, almost periodic or almost automorphic when the forcing term is periodic, almost periodic or almost automorphic, respectively. An alternative class of functions extending periodic and almost periodic functions which has the property that a bounded temporally global solution solution of a nonautonomous ordinary differential equation belongs to this class when the forcing term does is introduced here. Specifically, the class of functions consists of uniformly continuous functions, defined on the real line and taking values in a Banach space, which have pre-compact ranges. Besides periodic and almost periodic functions, this class also includes many nonrecurrent functions. Assuming a hyperbolic structure for the unperturbed linear equation and certain properties for the linear and nonlinear parts, the existence of a special bounded entire solution, as well the existence of stable and unstable manifolds of this solution are established. Moreover, it is shown that this solution and these manifolds inherit the temporal behaviour of the vector field equation. In the stable case it is shown that this special solution is the pullback attractor of the system. A class of infinite dimensional examples involving a linear operator consisting of a time independent part which generates a $\mathcal{C}_0$-semigroup plus a small time dependent part is presented and applied to systems of coupled heat and beam equations.


## 1. Introduction

A classical problem in the qualitative theory of ordinary differential equations (ODEs) is to determine if and when a bounded temporally global solution, of a nonautonomous ODE, if one exists, has the same temporal behaviour as the vector field, see for example [1–5]. The periodic and almost periodic cases have received extensive attention, motivated by mechanics and, especially, by celestial mechanics. In particular, it is known that when such a periodic or almost periodic solution exists, then its module is contained in that of the vector field, which says essentially that the temporal behaviour of the solution is subsumed in that of the vector field, hence is no more complicated.

Such bounded entire solutions, however, need not exist at all and when they do exist they need not have the same temporal behaviour as the vector field. Johnson [6] gave example of a linear ODE with an almost periodic coefficient matrix which has a nontrivial almost automorphic solution that is not almost periodic. The dynamics of ODEs with almost automorphic forcing terms has been discussed extensively, see for example [7–10]. (See [8] for a simple example of an almost automorphic function which is not almost periodic.)

☆ Partially supported by the Ministerio de Ciencia e Innovación (Spain) grant MTM2008-00088 and Junta de Andalucía grant P07-FQM-02468.2 (Spain), CAPES/DGU 267/2008 (Brazil/Spain), Processo Fapesp 2008/53317-3, Processo CNPq 301881/2008-1 and CAPES-DAAD (Brazil/Germany)
* Corresponding author. Fax: +49 69 698 28846.
*E-mail addresses:* kloeden@math.uni-frankfurt.de (P.E. Kloeden), hmr@icmc.usp.br (H.M. Rodrigues).

0362-546X/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2010.12.025