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Strong full bounded solutions of nonlinear parabolic equations with nonlinear boundary conditions

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ABSTRACT

We study the existence of (generalized) bounded solutions existing for all times for nonlinear parabolic equations with nonlinear boundary conditions on a domain that is bounded in space and unbounded in time (the entire real line). We give a counterexample which shows that a (weak) maximum principle does not hold in general for linear problems defined on the entire real line in time. We consider a boundedness condition at minus infinity to establish (one-sided) L^∞-*a priori* estimates for solutions to linear boundary value problems and derive a weak maximum principle which is valid on the entire real line in time. We then take up the case of nonlinear problems with (possibly) nonlinear boundary conditions. By using comparison techniques, some (delicate) *a priori* estimates obtained herein, and nonlinear approximation methods, we prove the existence and, in some instances, positivity and uniqueness of strong full bounded solutions existing for all times.

1. Introduction

Consider the nonlinear parabolic boundary value problem

$$\begin{cases} \dfrac{\partial u}{\partial t}(x,t) - Lu(x,t) = f(x,t,u) & \text{a.e. in } \Omega \times \mathbb{R}, \\ \mathcal{B}u = \varphi(x,t,u) & \text{a.e. on } \partial\Omega \times \mathbb{R}, \\ \sup\limits_{\Omega\times\mathbb{R}} |u(x,t)| < \infty, & \end{cases} \tag{1.1}$$

where Ω is a bounded, open and connected subset of $\mathbb{R}^N$ with boundary $\partial\Omega$ and closure $\overline{\Omega}$. We suppose that L is a second-order, uniformly elliptic differential operator with time-dependent coefficients and $\mathcal{B}$ is a linear first-order boundary operator which is of either Dirichlet, Neumann, or regular oblique type. We suppose that the coefficients of the operators L and $\mathcal{B}$ are, say, measurable and bounded. The reaction and the boundary nonlinearities f and φ are, say, *Carathéodory* functions.

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