Nonlinear Analysis 74 (2011) 195–212

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Well-posedness, stability and invariance results for a class of multivalued Lur'e dynamical systems

Bernard Brogliato [a,*], Daniel Goeleven [b]

[a] INRIA, BIPOP project-team, ZIRST Montbonnot, 655 avenue de l'Europe, 38334 Saint Ismier, France

[b] PIMENT, Université de La Réunion, Saint-Denis, 97400, France

ARTICLE INFO

Article history:
Received 16 December 2009
Accepted 19 August 2010

ABSTRACT

This paper analyzes the existence and uniqueness issues in a class of multivalued Lur'e systems, where the multivalued part is represented as the subdifferential of some convex, proper, lower semicontinuous function. Through suitable transformations the system is recast into the framework of dynamic variational inequalities and the well-posedness (existence and uniqueness of solutions) is proved. Stability and invariance results are also studied, together with the property of continuous dependence on the initial conditions. The problem is motivated by practical applications in electrical circuits containing electronic devices with nonsmooth multivalued voltage/current characteristics, and by state observer design for multivalued systems.

1. Introduction

Lur'e systems, which consist of a linear time-invariant system in negative feedback with a static nonlinearity satisfying a sector condition, have received a considerable interest in the applied mathematics and control literature, due to their broad interest (see [1] for a survey). More recently the case where the nonlinearity is a maximal monotone map has been studied [2]. The maximal monotonicity allows one to consider unbounded sectors $[0, +\infty]$ and nonsmooth set-valued nonlinearities. So-called linear complementarity systems can be recast into Lur'e systems, where the feedback nonlinearity takes the form of a set of complementarity conditions between two slack variables [3–5]. One of these slack variables may be interpreted as a Lagrange multiplier λ, while the other one usually takes the form $y = Cx + D\lambda$. More general piecewise linear nonlinearities have been considered in [6,7]. As pointed out in [2] there exists a close relationship between some complementarity systems and differential inclusions with maximal monotone right-hand sides, in particular inclusions into normal cones to convex sets (which are in turn equivalent to dynamical variational inequalities of the first kind). Particular cases have been investigated in [8–10]. All these works are however restricted to the case where $D = 0$, except [7] where affine complementarity systems are considered. In this paper, we extend the works in [8,9] to the case where $D \neq 0$, i.e. there exists a feedthrough matrix in the linear part of the system. Moreover the nonlinearities which we consider are much more general than complementarity conditions between y and λ (i.e. $y \geq 0$, $\lambda \geq 0$, $y^T\lambda = 0$) and the considered systems may be written equivalently as dynamical variational inequalities of the second kind. Such an extension may be important in practice (for instance electrical circuits with ideal diodes and transistors usually yield systems with a nonzero feedthrough matrix D, possibly positive semi-definite and non-symmetric). Observer synthesis for set-valued systems is also an important application [11,12]. This work may also be seen as the continuation of previous efforts to study the relationships

* Corresponding author. Tel.: +33 476615393; fax: +33 476615252.
E-mail addresses: Bernard.Brogliato@inrialpes.fr (B. Brogliato), Daniel.Goeleven@univ-reunion.fr (D. Goeleven).

0362-546X/$ – see front matter
doi:10.1016/j.na.2010.08.034