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On a new Kato class and positive solutions of Dirichlet problems for the fractional Laplacian in bounded domains

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ABSTRACT

The purpose of this paper is to extend some results of the potential theory of an elliptic operator to the fractional Laplacian $(-\Delta)^{\alpha/2}, 0 < \alpha < 2$, in a bounded $C^{1,1}$ domain D in $\mathbb{R}^n$. In particular, we introduce a new Kato class $K_\alpha(D)$ and we exploit the properties of this class to study the existence of positive solutions of some Dirichlet problems for the fractional Laplacian.

1. Introduction

This paper deals with one of the most important subfamilies of Levy processes called symmetric stable processes. A symmetric α-stable process $(X_t)_{t>0}$ on $\mathbb{R}^n$ is a Levy process whose transition density $p(t, x-y)$ relative to the Lebesgue measure is uniquely determined by its Fourier transform

$$\int_{\mathbb{R}^n} \exp(\mathrm{i}x.\xi)p(t,x)\mathrm{d}x = \exp(-t\,|\xi|^\alpha).$$

The constant α must be in $(0, 2]$, and we always assume that $n \geq 2$. When $\alpha = 2$, we get a Brownian motion running with a time clock twice as fast as the standard one. In this paper, we only consider $0 < \alpha < 2$ and compare our results to corresponding ones in the classical case $\alpha = 2$.

Recently, there has been intense interest in studying stable processes, due to their applications in many branches of sciences such as physics, operations research, queuing theory, mathematical finance, and risk estimation. For these and more applications of stable processes, please see the interesting book [1], and the article [2] and the references therein.

There is significant difference between Brownian motion and a symmetric α-stable process. In fact, unlike Brownian motion, the symmetric α-stable process $(X_t)_{t>0}$ has discontinuous sample paths. Its Levy measure is given by

$$\nu(\mathrm{d}x) = c_{n,\alpha}\,|x|^{-(n+\alpha)}\,\mathrm{d}x,$$

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