Nonlinear Analysis 74 (2011) 3242–3260

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Variational approach and optimal control of a PEM fuel cell

A. Căpăţînă [a], H. Ene [a,b], G. Paşa [a], D. Poliševski [a,b], R. Stavre [a,*]

[a] *Institute of Mathematics "Simion Stoilow", Romanian Academy, P.O. Box 1-764, 014700 Bucharest, Romania*
[b] *University of Piteşti, Piteşti, Romania*

ARTICLE INFO

Article history:
Received 24 February 2010
Accepted 5 February 2011

MSC:
49J20
49K20
76S05
80A20

Keywords:
PEM fuel cell
A priori regularity
Penalization
Boundary control
Optimality conditions

ABSTRACT

The purpose of this paper is to propose and study a mathematical model and a boundary control problem associated to the miscible displacement of hydrogen through the porous anode of a PEM fuel cell. Throughout the paper, we study certain variational problems with *a priori* regularity properties of the weak solutions. We obtain the existence of less regular solutions and then we prove the desired regularity of these solutions. We consider a control problem that permits to determine the boundary distribution of the pressure which provides an optimal configuration for the temperature and for the concentration, as well. Since the solution of the problem is not unique, the control variable does not appear explicitly in the definition of our cost functional. To overcome this difficulty, we introduce a family of penalized control problems which approximates our boundary control problem. The necessary conditions of optimality are derived by passing to the limit in the penalized optimality conditions.

1. Introduction

For its versatility the PEM fuel cell is one of the most interesting new energy sources. In the recent years, a significant number of works have presented the physical and chemical processes involved in the fluid motion through a PEM fuel cell (see e.g. [1–9]). By [10,11] we started our approach to the mathematical problems issued from the study of certain processes taking place in a PEM fuel cell, such as: the miscible displacement of the sulphuretted hydrogen, the oxidation process, the heat and mass transport.

A PEM fuel cell consists of several layers of basic cells. In these cells, a proton conductivity membrane separates the anode and the cathode side, and each side has an electrode. The anode of the fuel cell is considered a porous medium occupying a bounded domain Ω. The absorption of hydrogen is made through a part of the boundary of the porous medium. During the oxidation process, in the presence of an electrolyte, hydrogen dissociates into protons and electrons; the protons are conducted through the membrane of the cathode, whereas the electrons provide electric energy in an external circuit. At the cathode, the electrons and positively charged hydrogen ions combine with oxygen to form water, which flows out of the cell. During this process, water, heat and electric energy are produced.

The present paper is devoted to the study of a mathematical model and of a boundary control problem associated to the miscible displacement of hydrogen through the porous anode of a PEM fuel cell. We are taking into account the miscible displacement of hydrogen through the porous anode of the fuel cell, the oxidation process and the heat and mass transport. We assume that the process is governed by a nonlinear system containing the continuity equation, the Darcy's law, the diffusion equation and the heat equation. Also, the variation of the temperature and concentration are supposed small

* Corresponding author.
E-mail address: ruxandra.stavre@imar.ro (R. Stavre).

0362-546X/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2011.02.003