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Existence and uniqueness analysis of a detached shock problem for the potential flow

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ABSTRACT

We study a problem for two-dimensional steady potential and isentropic Euler equations in a bounded domain, where an artificial detached shock interacts with a wedge. Using the stream function, we obtain a free boundary problem for the subsonic state and the detached artificial shock curve and we prove that such configuration admits a unique solution in certain weighted Hölder spaces. The proof is based on various Hölder and Schauder estimates for second-order elliptic equations and fixed point theorems. Moreover, we pose an energy principle and remark that the physical attached shock is the minimizer of the energy functional.

1. Introduction

We consider the two-dimensional steady Euler equations that consist of the equation of conservation of mass

$$(\rho u)_x + (\rho v)_y = 0, \tag{1.1}$$

and Bernoulli's law

$$\frac{q^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p}{\rho} = \frac{1}{\gamma - 1}, \tag{1.2}$$

where $\rho, u, v : \mathbb{R}^2 \to \mathbb{R}$ stand for density and velocity in the x and y directions, respectively, $p : \mathbb{R}^2 \to \mathbb{R}$ denotes pressure, $q^2 = u^2 + v^2$ and $\gamma > 1$ is the gas constant. We assume that the flow is isentropic and potential, that is the pressure is given by the equation

$$p(\rho) = \frac{\rho^\gamma}{\gamma}, \tag{1.3}$$

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