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Initial value problems for creeping flow of Maxwell fluids

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ABSTRACT

We consider the flow of nonlinear Maxwell fluids in the unsteady quasistatic case, where the effect of inertia is neglected. We study the well-posedness of the resulting PDE initial-boundary value problem locally in time. This well-posedness depends on the unique solvability of an elliptic boundary value problem. We first present results for the 3D case with sufficiently small initial data and for a simple shear flow problem with arbitrary initial data; after that we extend our results to some 3D flow problems with large initial data.

We solve our problem using an iteration between linear subproblems. The limit of the iteration provides the solution of our original problem.

1. Introduction

Viscoelastic fluids, such as egg whites, liquid soap, liquid polymers, oils, etc., are classified as non-Newtonian fluids with memory. That is, in addition to the current velocity field, the extra stress also depends on the history of previous deformations [1–3]. For linear viscoelasticity, Maxwell proposed a linear constitutive model by connecting in series linear viscous and elastic elements [2,4]. The mathematical representation for this model is a linear differential equation relating the extra stress tensor with the velocity gradient. Many widely used models are based on adding nonlinear terms to the Maxwell model. We shall refer to these models as Maxwell-type differential constitutive equations [2].

Our goal in this paper is to study the basic properties of the governing equations, which have the unknown variables of extra stress tensor, velocity field and pressure. We shall examine the well-posedness for an unsteady flow problem of a quasistatic viscoelastic Maxwell fluid, i.e. the fluid acceleration effect is neglected. To prove the existence of the solution, we use an iterative method introduced by Renardy in [5,6,1]. This method transforms our problem to iterative linear subproblems. At each iteration we solve an elliptic problem and two hyperbolic problems. The limit of the iteration provides the solution of our original problem. Here we present the results for the following cases:

- Three-dimensional flow, in a bounded domain, globally in time for sufficiently small initial data.
- Simple shear flow, i.e. a spatially one-dimensional problem, locally in time with arbitrary initial data.
- Some three-dimensional flow problems, in a bounded domain, locally in time with large initial data.

We note that results analogous to ours for the case where inertial terms are included are well known, see [7–10]. It might seem that the quasistatic case should be easier; in fact some of the very first existence results for initial value problems in viscoelasticity were for quasistatic deformation of viscoelastic solids [11]. However, for three-dimensional problems with large initial data, the local well-posedness of the initial value problem depends on the ellipticity and unique solvability of a nonlinear system of PDEs which determine the initial velocity. While conditions for ellipticity are relatively easy to analyze, the unique solvability poses a challenge which, to our knowledge, remains unsolved for any model of viscoelastic flow. In our analysis in Section 6, we shall assume spatially uniform initial stresses; in this case, unique solvability follows from ellipticity.

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