Nonlinear Analysis 74 (2011) 462–478

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

On a problem of magnetohydrodynamics in a multi-connected domain

Sunra J.N. Mosconi [a,*], Vsevolod A. Solonnikov [b]

[a] University of Catania, Italy
[b] University of St. Petersburg, Russian Federation

ARTICLE INFO

Article history:
Received 4 February 2010
Accepted 1 September 2010

Keywords:
Magnetohydrodynamics
Navier–Stokes equations
Stability
Hodge decomposition
Maxwell equations

ABSTRACT

We consider the following problem in the MHD approximation: the vessel $\Omega_1 \subset \Omega$ is filled with an incompressible, electrically conducting fluid, and is surrounded by a dielectric or by vacuum, occupying the bounded domain $\Omega_2 = \Omega \setminus \Omega_1$. In Ω we have a magnetic and electric field and the external surface $S = \partial\Omega$ is an ideal conductor. The emphasis in the paper is on when Ω is not simply connected, in which case the MHD system is degenerate. We use Hodge-type decomposition theorems to obtain strong solutions locally in time or global for small enough initial data, and a linearization principle for the stability of a stationary solution.

1. Introduction

Given two bounded smooth domains Ω_1 and Ω in three-dimensional space, with $\Omega_1 \Subset \Omega$, we consider the following problem.

The vessel Ω_1 is filled with an incompressible, electrically conducting fluid, and is surrounded by a dielectric or by vacuum, occupying the bounded domain $\Omega_2 = \Omega \setminus \Omega_1$. We have a magnetic and electric field in Ω and the external surface $S = \partial\Omega$ is an ideal conductor. We assign in Ω_1 an external hydrodynamic force density $\boldsymbol{f}$ and a current density $\boldsymbol{j}$, and we assign at time $t = 0$ the initial velocity and magnetic field. We determine the motion of the fluid.

We will assume the quasi-stationary approximation, finite conductivity σ of the fluid and ignore the Hall effect. In this setting the equations of magnetohydrodynamics in Ω_1 have the form

$$\boldsymbol{v}_t - \nu\Delta\boldsymbol{v} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} - \mu(\boldsymbol{H}\cdot\nabla)\boldsymbol{H} + \nabla\left(p + \mu\frac{|\boldsymbol{H}|^2}{2}\right) = \boldsymbol{f}, \tag{1}$$

$$\mu\boldsymbol{H}_t = -\mathrm{rot}\boldsymbol{E}, \qquad \mathrm{rot}\boldsymbol{H} = \sigma(\boldsymbol{E} + \mu(\boldsymbol{v}\times\boldsymbol{H})) + \boldsymbol{j} \tag{2}$$

$$\mathrm{div}\boldsymbol{v} = 0, \qquad \mathrm{div}\boldsymbol{H} = 0 \tag{3}$$

where $\boldsymbol{v}$ and p are the velocity and pressure of the liquid, $\boldsymbol{H}$ and $\boldsymbol{E}$ are the magnetic and electric vector fields respectively, $\mu, \nu > 0$ are the magnetic permeability and the viscosity of the fluid and $\sigma > 0$ is the electric permeability. For when the fluid and the dielectric (or vacuum) have different magnetic permeabilities, respectively μ_1 and μ_2, we will set $\mu(x) \equiv \mu_1$ in Ω_1 and $\mu(x) \equiv \mu_2$ in Ω_2 Eliminating the electric field from (2) one obtains in Ω_1

$$\boldsymbol{H}_t + \eta\,\mathrm{rot}\mathrm{rot}\boldsymbol{H} - \mathrm{rot}(\boldsymbol{v}\times\boldsymbol{H}) = \eta\,\mathrm{rot}\boldsymbol{j}, \qquad \mathrm{div}\boldsymbol{H} = 0, \tag{4}$$

where $\eta = 1/\sigma\mu_1$ is the magnetic diffusivity of the fluid.

* Corresponding author. Tel.: +39 3281271649.
E-mail addresses: mosconi@dmi.unict.it, mosconi@cibs.sns.it (S.J.N. Mosconi), solonnik@pdmi.ras.ru (V.A. Solonnikov).

0362-546X/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2010.09.002