Nonlinear Analysis 74 (2011) 4455–4473

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Spectrum of some integro-differential operators and stability of travelling waves

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ARTICLE INFO

Article history:
Received 14 February 2010
Accepted 2 April 2011
Communicated by Ravi Agarwal

MSC:
34K08
35K57

Keywords:
Spectral properties
Nonlocal elliptic operator
Travelling waves
Stability

ABSTRACT

Spectral properties of some integro-differential operators on $\mathbb{R}^1$ are studied. Characterisation of the principal eigenvalue is obtained in terms of the positive eigenfunction. These results are used to prove local and global stability of travelling waves and to find their speed.

1. Introduction

In this paper we shall study some spectral properties of the linear integro-differential operator

$$Lu = u'' + cu' + a(x)\phi * u + b(x)u, \tag{1.1}$$

acting from the Banach space $E = C_0^2(\mathbb{R})$ into the Banach space $F = C_0(\mathbb{R})$. Here $C_0(\mathbb{R})$ denotes the space of continuous functions from $\mathbb{R}$ into itself that tend to zero at $x = \pm\infty$ while E is the space of functions u of class C^2 such that u, u' and u'' belong to F. The functions $x \to a(x)$ and $x \to b(x)$ are supposed to be bounded and continuous with the limits $a^{\pm}$ and $b^{\pm}$ when $x \to \pm\infty$, respectively, while $\phi * u$ denotes the convolution product,

$$\phi * u(x) = \int_{-\infty}^{\infty} \phi(x-y)u(y)\mathrm{d}y \quad x \in \mathbb{R},$$

where $\phi(x)$ is some integrable function.

The spectrum of the operator L consists of its essential part and of eigenvalues. Here and in the sequel, the essential spectrum is understood as the Fredholm spectrum, namely the set of complex λ for which the operator $L - \lambda$ does not satisfy the Fredholm property.

On the one hand, the location of the essential spectrum is an important point when dealing with some nonlinear problems. Indeed, most of the tools of nonlinear analysis, such as the implicit function theorem, bifurcation analysis or

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0362-546X/$ – see front matter
doi:10.1016/j.na.2011.04.009