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Blow-up of rough solutions to the fourth-order nonlinear Schrödinger equation[☆]

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ABSTRACT

This paper deals with the formation of singularities of rough blow-up solutions to the fourth-order nonlinear Schrödinger equation. The limiting profile and L^2-concentration of the rough blow-up solutions are obtained in $H^s(\mathbb{R}^4)$ with $s > s_0$, where $s_0 \leq \frac{9+\sqrt{721}}{20} \approx 1.793$. The new ingredient relies on the refined compactness result developed by Zhu et al. [S.H. Zhu, J. Zhang, H. Yang, Limiting profile of the blow-up solutions for the fourth-order nonlinear Schrödinger equation, Dyn. Partial Differ. Equ. 7 (2010) 187–205].

1. Introduction

In this paper, we study the Cauchy problem of the following fourth-order nonlinear Schrödinger equation

$$iu_t - \Delta^2 u + |u|^2 u = 0, \quad t \geq 0,\ x \in \mathbb{R}^4, \tag{1.1}$$

$$u(0,x) = u_0, \tag{1.2}$$

where $i = \sqrt{-1}$; $\Delta^2 = \Delta\Delta$ is the biharmonic operator defined in $\mathbb{R}^4$ and $\Delta = \sum_{j=1}^{4} \frac{\partial^2}{\partial x_j^2}$ is the Laplace operator in $\mathbb{R}^4$; $u = u(t,x)$: $[0,T^*) \times \mathbb{R}^4 \to \mathbb{C}$ is the complex valued function and $0 < T^* \leq +\infty$. Fourth-order Schrödinger equations are introduced by Karpman [1], Karpman and Shagalov [2] to take into account the role of small fourth-order dispersion terms in the propagation of intense laser beams in a bulk medium with Kerr nonlinearity, and such fourth-order Schrödinger equations are written as

$$i\phi_t + \varepsilon\Delta^2\phi + \mu\Delta\phi + |\phi|^{p-1}\phi = 0, \qquad \phi = \phi(t,x) : I \times \mathbb{R}^d \to \mathbb{C}, \tag{1.3}$$

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