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Traveling waves in a nonlocal dispersal population model with age-structure[☆]

Guo-Bao Zhang

School of Mathematics and Statistics, Lanzhou University, Lanzhou, Gansu 730000, People's Republic of China

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ABSTRACT

This paper is concerned with the traveling waves in a single species population model which is derived by considering the nonlocal dispersal and age-structure. If the birth function is monotone, then the existence of traveling wavefront is reduced to the existence of a pair of super and subsolutions without the requirement of smoothness. It is proved that the traveling wavefront is strictly increasing and unique up to a translation. The asymptotic behavior of traveling wavefronts is also obtained. If the birth function is not monotone, the existence of traveling wave solution is affirmed by introducing two auxiliary nonlocal dispersal equations with quasi-monotonicity.

1. Introduction

In population dynamics, many phenomena can be described by reaction–diffusion equations with time delay; see [1–4]. Traveling wave solution is one of the most important solutions for these equations due to its important role in determining the long term behavior of solutions for the corresponding Cauchy problems and describing the propagation of patterns and domain invasion of species; see [5–13] and the references therein.

In view of the interaction of time delay and spatial diffusion, namely, the individuals may be at different locations in the history, the so-called delay induced nonlocality occurs. For example, So et al. [14] derived a model to formulate the evolution of the matured population of a single species with age-structure and living in a spatially unbounded environment, and the model is the following reaction–diffusion equation with time delay and nonlocal effect

$$\frac{\partial w(x,t)}{\partial t} = D\Delta w - dw(x,t) + \epsilon \int_{\mathbb{R}} b\big(w(y,t-\tau)\big) f_\alpha(x-y)\mathrm{d}y, \tag{1.1}$$

where $w(x,t)$ represents the matured population density of the species at position x and at time t. $D > 0$ and $d > 0$ denote the diffusion rate and death rate of the matured population, respectively. $\tau \geq 0$ is the maturation time, $b(\cdot)$ models the birth function, $f_\alpha(x) = \frac{1}{\sqrt{4\pi\alpha}} e^{-x^2/4\alpha}$, $\epsilon > 0$ and $\alpha \geq 0$ reflect the impact of the death rate and the diffusion rate of the immature on the matured population, respectively.

For Eq. (1.1) in the whole $\mathbb{R}$, there are a lot of research works on the existence, uniqueness and stability of traveling waves. For example, see [14,15,6,9,12,8] for the monostable nonlinearity, and [16] for the bistable nonlinearity.

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E-mail addresses: zhanggb2008@lzu.edu.cn, zhanggb2008@lzu.cn.

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