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Local well-posedness for the homogeneous Euler equations ☆

Xin Zhong, Xing-Ping Wu *, Chun-Lei Tang

School of Mathematics and Statistics, Southwest University, Chongqing 400715, People's Republic of China

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ABSTRACT

We introduce Triebel–Lizorkin–Lorentz function spaces, based on the Lorentz $L^{p,q}$-spaces instead of the standard L^p-spaces, and prove a local-in-time unique existence and a blow-up criterion of solutions in those spaces for the Euler equations of inviscid incompressible fluid in $\mathbb{R}^n$, $n \geq 2$. As a corollary we obtain global existence of solutions to the 2*D* Euler equations in the Triebel–Lizorkin–Lorentz space. For the proof, we establish the Beale–Kato–Majda type logarithmic inequality and commutator estimates in our spaces. The key methods of proof used are the Littlewood–Paley decomposition and the paradifferential calculus by J.M. Bony.

1. Introduction and main results

In this paper, we consider the Euler equations for the inviscid incompressible fluid in $\mathbb{R}^n$, $n \geq 2$,

$$\begin{cases} \partial_t v + (v \cdot \nabla) v = -\nabla p, & (x,t) \in \mathbb{R}^n \times (0,\infty), \\ \operatorname{div} v = 0, & (x,t) \in \mathbb{R}^n \times (0,\infty), \\ v(x,0) = v_0(x), & x \in \mathbb{R}^n, \end{cases} \tag{1.1}$$

where $v = (v_1, v_2, \dots, v_n)$, $v_j = v_j(x,t)$, $j = 1, 2, \dots, n$, is the velocity of the fluid flows, $p = p(x,t)$ is the scalar pressure, and v_0 is the given initial velocity satisfying $\operatorname{div} v_0 = 0$.

One of the outstanding open questions in mathematical fluid dynamics today is whether the incompressible three-dimensional Euler equations develop a singularity in the vorticity field in a finite time. The interest in singularities comes from many directions. Physically their formation may signify the onset of turbulence and may be a mechanism for energy transfer to small scales. Numerically they require very special methods and are thus a challenge to computational fluid dynamics. Finally, the question is of interest to mathematicians because of the question of global existence of solutions.

For the local-in-time existence and uniqueness of solutions for the Euler equations, there are many results. Given $v_0 \in H^m(\mathbb{R}^n)$ for integer $m > \frac{n}{2} + 1$, Kato [1] proved local-in-time existence and uniqueness of a solution in the class $C([0,T]; H^m(\mathbb{R}^n))$, where $T = T(\|v_0\|_{H^m})$. Kato and Ponce [2] extended this result to the fractional-order Sobolev space $W^{s,p}(\mathbb{R}^n) = (1-\Delta)^{-\frac{s}{2}} L^p(\mathbb{R}^n)$ for $s > \frac{n}{p} + 1$, $1 < p < \infty$. Furthermore, Lichtenstein established local existence in the Hölder space $C^{1,\gamma}(\mathbb{R}^n)$, Chemin [3] gave another local existence proof in $\mathbb{R}^n$. Moreover, a number of studies on the Euler equations in Besov spaces $B^s_{p,r}(\mathbb{R}^n)$ has been done by Vishik [4–6], Chae [7], Zhou [8,9], Zhou et al. [10].

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* Corresponding author. Tel.: +86 23 68253135.
E-mail address: wuxp@swu.edu.cn (X.-P. Wu).

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