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A comparative study of approximate symmetry and approximate homotopy symmetry to a class of perturbed nonlinear wave equations

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ABSTRACT

A comparative study of approximate symmetry and approximate homotopy symmetry to a class of perturbed nonlinear wave equations is performed. First, complete infinite-order approximate symmetry classification of the equation is obtained by means of the method originated by Fushchich and Shtelen. An optimal system of one-dimensional subalgebras is derived and used to construct general formulas of approximate symmetry reductions and similarity solutions. Second, we study approximate homotopy symmetry of the equation and construct connections between the two symmetry methods for the first-order and higher-order cases, respectively. The series solutions derived by the two methods are compared.

1. Introduction

Symmetry methods for differential equations, originally developed by Sophus Lie, have been evolved into one of the most explosive developments of mathematics and physics throughout the past century. There have been considerable important generalizations in this method which include nonclassical symmetry, Lie–Bǎcklund symmetry, potential symmetry, etc. [1–6]. Usually, with a continuous differential equation, we can study its invariance, symmetry properties and similarity reductions by means of the Lie symmetry method [1,3]. In particular, for the mathematical models described by differential equations containing arbitrary elements (parameters or functions) which have been found experimentally and so are not strictly fixed, the symmetry approach allows one to simplify them which make the models admit a symmetry group with certain properties or the most extensive symmetry group [7–9].

It is well known that there exist differential equations of physical interest with a small parameter possessing few exact symmetries or none at all and even if exist, the small parameter also disturbs symmetry group properties of the unperturbed equation [10,11]. Hence, two methods were introduced to study approximate symmetry of this type of equations. The first method due to Baikov et al. [10,11] represents a perturbation technique embedded into the standard procedure of the classical Lie symmetry method, which implements perturbation for symmetry generators. In 1989, Fushchich and Shtelen [12] proposed the second method which expand the dependent variables in terms of a small parameter (may be a physical parameter or artificially introduced) as the usual perturbation analysis and the method was later followed by Euler et al. [13–15]. In [16,17], two methods are applied to several equations and the comparisons are discussed.

Later, Liao [18,19] introduced the homotopy analysis method, which is a combination of the classical perturbation technique and homotopy concept as used in topology, to get series solutions of various types of nonlinear problems. In this method, the solution is considered as the sum of an infinite series, which converges rapidly to accurate solutions of the

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