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Linearly perturbed polyhedral normal cone mappings and applications ☆

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ABSTRACT

Under a mild regularity assumption, we derive an exact formula for the Fréchet coderivative and some estimates for the Mordukhovich coderivative of the normal cone mappings of perturbed polyhedra in reflexive Banach spaces. Our focus point is a *positive linear independence condition*, which is a relaxed form of the *linear independence condition* employed recently by Henrion et al. (2010) [1], and Nam (2010) [3]. The formulae obtained allow us to get new results on solution stability of affine variational inequalities under linear perturbations. Thus, our paper develops some aspects of the work of Henrion et al. (2010) [1] Nam (2010) [3] Qui (in press) [12] and Yao and Yen (2009) [6,7].

1. Introduction

Consider a real Banach space X with the dual denoted by X^*, a finite index set $T = \{1, 2, \ldots, m\}$, a vector system $\{a_i^* \in X^* \mid i \in T\}$, and a polyhedral convex set (a *polyhedron*, for brevity)

$$\Theta(b) = \{x \in X \mid \langle a_i^*, x\rangle \leq b_i \text{ for all } i \in T\}. \tag{1.1}$$

Here $b := (b_1, \ldots, b_m) \in \mathbb{R}^m$ is a parameter. We interpret $b_1, \ldots, b_m$ as *right-hand side perturbations* of the linear inequality system

$$\langle a_i^*, x\rangle \leq b_i, \quad i \in T. \tag{1.2}$$

The *active index set* corresponding to a pair (x, b), where $x \in \Theta(b)$, is defined by

$$I(x, b) = \{i \in T \mid \langle a_i^*, x\rangle = b_i\}. \tag{1.3}$$

For a subset $I \subset T$, put $\bar{I} = T \setminus I$. By b_I (resp., $b_{\bar{I}}$) we denote the vector with the components b_i where $i \in I$ (resp., $i \in \bar{I}$).

The two-variable multifunction $\mathcal{F} : X \times \mathbb{R}^m \rightrightarrows X^*$,

$$\mathcal{F}(x, b) := N(x; \Theta(b)) \quad \forall (x, b) \in X \times \mathbb{R}^m, \tag{1.4}$$

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