



A parabolic variational inequality related to the perpetual American executive stock options

Song Liping^{a,b,c}, Yu Wanghui^{a,b,*}

^a School of Mathematic Science, Soochow University, Suzhou 215006, China

^b Research Center of Financial Engineering, Soochow University, Suzhou 215006, China

^c Department of Mathematics, Putian University, Putian 351100, China

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ABSTRACT

A parabolic variational inequality is investigated which comes from the study of the optimal exercise strategy for the perpetual American executive stock options in financial markets. It is a degenerate parabolic variational inequality and its obstacle condition depends on the derivative of the solution with respect to the time variable. The method of discrete time approximation is used and the existence and regularity of the solution are established.

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1. Introduction

Executive stock options (ESOs) are call options on the stock of a firm granted to an executive by the firm as part of his or her remuneration package. These options bestow the right to the executive to buy the stocks of the firm at a pre-arranged price on pre-arranged days. They encourage the executive to work towards an increase in the financial health of the firm, which will increase the share price of the firm, and eventually increase his or her own wealth.

Since the mid-1980s, executive stock options have become an important component of compensation to the executives in the United State and other countries. However, ESOs are an expense to the firm because the firm is buying services from the executive. The ESOs issued by a firm are so large that the cost to the firm should not be ignored. In 2004, under Statement of Financial Accounting Standards No. 123 (revised), the Financial Accounting Standards Board required firms to account for their costs of ESOs. In order to calculate the firm's cost of ESOs, it is necessary to have rational prediction of the future exercise strategies of the executives since the ESOs usually have long maturity ranging from 5 to 15 years. This gives rise to the need to create a reasonable valuation method for ESOs.

Executive stock options are a kind of American call option (i.e., they can be exercised at any time during the exercise window), with long maturity ranging from 5 to 15 years. However, they differ from the vanilla American call options in several crucial ways, for example, they cannot be transferred when the executive leaves the company and they cannot be traded in the option market. The executive's risk preference also affects his or her exercise behavior and hence the value

* Corresponding author at: School of Mathematic Science, Soochow University, Suzhou 215006, China. Tel.: +86 13815250291.

E-mail addresses: lipingsong@126.com (L. Song), whyu@suda.edu.cn (W. Yu).