Nonlinear Analysis 74 (2011) 4433–4447

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Existence and properties of solutions of a control system with hysteresis effect☆

S.A. Timoshin, A.A. Tolstonogov*

Institute for System Dynamics and Control Theory, Siberian Branch, Russian Academy of Sciences, Lermontov Str., 134, Irkutsk, 664033, Russia

ARTICLE INFO

Article history:
Received 28 December 2010
Accepted 1 April 2011
Communicated by Ravi Agarwal

Keywords:
Evolution control systems
Subdifferential
Nonconvex constraints
Extreme points
Mosco convergence
Hysteresis

ABSTRACT

We consider a control system described by two ordinary nonlinear differential equations subject to a control constraint given by a multivalued mapping with closed nonconvex values, which depends on the phase variables. One of the equations contains the subdifferential of the indicator function of a closed convex set depending on the unknown phase variable. The equation containing the subdifferential describes an input–output relation of hysteresis type.

Along with the original control constraint, we also consider the convexified control constraint and the constraint consisting of the extremal points of the convexified control constraint.

We prove the existence of solutions of our control system with various control constraints and establish certain relationships between corresponding solution sets.

1. Introduction

Consider a nonlinear control system described by two ordinary differential equations of the following form

$$a_1(v(t), w(t))v'(t) + a_2(v(t), w(t))w'(t) = g(v(t), w(t))u^1(t) + c_1(v(t), w(t)), \tag{1.1}$$

$$b_1(v(t), w(t))v'(t) + b_2(v(t), w(t))w'(t) + \partial I_{v(t)}(w(t)) \ni h(v(t), w(t))u^2(t) + c_2(v(t), w(t)), \tag{1.2}$$

$$v(0) = v_0, \qquad w(0) = w_0, \quad t \in T = [0, 1],$$

subject to the control constraint

$$u(t) = (u^1(t), u^2(t)) \in U(t, v(t), w(t)) \quad \text{a.e. on } T. \tag{1.3}$$

Here $a_i(\cdot,\cdot)$, $b_i(\cdot,\cdot)$, $c_i(\cdot,\cdot)$, $i = 1, 2$, $g(\cdot,\cdot)$, $h(\cdot,\cdot)$ are scalar functions; for each $v \in \mathbb{R}$, $\partial I_v(\cdot)$ is the subdifferential of the indicator function $I_v(\cdot)$ of the interval $[f_*(v), f^*(v)] \subset \mathbb{R}$ with $f_*(\cdot)$ and $f^*(\cdot)$ being two nondecreasing functions such that $f_* \leq f^*$ on $\mathbb{R}$; U is a multivalued mapping with closed bounded values; v_0 and w_0 are given numbers.

Along with (1.3) we will consider the following constraints

$$u(t) \in \operatorname{co} U(t, v(t), w(t)) \quad \text{a.e. on } T, \tag{1.4}$$

$$u(t) \in \operatorname{ext} \operatorname{co} U(t, v(t), w(t)) \quad \text{a.e. on } T, \tag{1.5}$$

☆ The research was supported by RFBR grant no. 10-01-00132 and by SB RAS (integration project no. 85, SB RAS - UrB RAS).

* Corresponding author. Tel.: +7 3952427100; fax: +7 3952511616.
E-mail addresses: sergey.timoshin@gmail.com (S.A. Timoshin), aatol@icc.ru (A.A. Tolstonogov).

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doi:10.1016/j.na.2011.04.004