



Nonlinear elliptic equations with Hardy potential and lower order term with natural growth[☆]

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ABSTRACT

In this work we analyze the interaction between the Hardy potential and a lower order term to obtain the existence or nonexistence of a positive solution in elliptic problems whose model is

$$\begin{cases} -\Delta_p u = g(u)|\nabla u|^p + \lambda \frac{u^{p-1}}{|x|^p} + f, & \text{in } \Omega, \\ u > 0, & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 3$, is a bounded domain containing the origin, $1 < p < N$ and Δ_p is the p -Laplacian. Concretely, we study under which range of values of the parameter $\lambda > 0$, the behavior of the positive continuous function g at infinity provides the existence of a solution for such a problem.

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1. Introduction

In this paper we deal with a class of nonlinear elliptic problems with a lower order term with natural growth with respect to the gradient and in particular its interaction with a nonlinear term singular at the origin.

More precisely, we deal with problems whose simplest model is

$$\begin{cases} -\Delta u = g(u)|\nabla u|^2 + \lambda \frac{u}{|x|^2} + f & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \quad (1.1)$$

where $\lambda \geq 0$, $\Omega \subset \mathbb{R}^N$, $N \geq 3$ and $0 \in \Omega$. We consider that f is a positive function in a suitable Lebesgue space and g is a nonnegative continuous function.

Our aim is to prove the existence and nonexistence of positive solutions for these kinds of Dirichlet problems and the summability of these solutions.

The difficulties in treating this problem arise from the presence of both the term with quadratic growth with respect to the gradient and the one singular at the origin, the so-called Hardy potential.

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