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Rates of convergence to zero for a semilinear parabolic equation with a critical exponent

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ABSTRACT

We study nonnegative solutions to the Cauchy problem for a semilinear parabolic equation with a nonlinearity which is critical in the sense of Joseph and Lundgren. We establish the rate of convergence to zero of solutions that start from initial data which are near the singular steady state. In the critical case, this rate contains a logarithmic term which does not appear in the supercritical case and makes the calculations more delicate.

1. Introduction

We consider the Cauchy problem

$$\begin{cases} u_t = \Delta u + u^p, & x \in \mathbb{R}^N,\ t \in (0,\infty), \\ u(x,0) = u_0(x), & x \in \mathbb{R}^N, \end{cases} \tag{1.1}$$

where $u = u(x,t)$, $p > 1$, Δ denotes the Laplacian operator with respect to x, and the function u_0 is nonnegative and continuous in $\mathbb{R}^N$. In spite of its simple structure, problem (1.1) offers a rich variety of mathematical phenomena and has been studied intensively by several authors. The monograph [1] and the references given therein provide a broad overview.

In particular, the large time behavior of solutions starting from initial data which are near the singular steady state has been studied by several authors. In order to present these results, we define the critical exponent

$$p_c := \begin{cases} \infty & \text{for } N \le 10, \\ \dfrac{(N-2)^2 - 4N + 8\sqrt{N-1}}{(N-2)(N-10)} & \text{for } N \ge 11, \end{cases}$$

which satisfies $p_c > \frac{N}{N-2} > 1$ for $N \ge 11$ and was found by Joseph and Lundgren (see [2]). Moreover, let $\varphi_\infty = \varphi_\infty(|x|)$ denote the singular steady state of (1.1), which exists for $p > \frac{N}{N-2}$, $N > 2$, and is given by

$$\varphi_\infty(|x|) := L|x|^{-m}, \quad |x| > 0,$$

where

$$m := \frac{2}{p-1} \quad \text{and} \quad L := \{m(N-2-m)\}^{\frac{1}{p-1}}.$$

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