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Bubbles with prescribed mean curvature: The variational approach

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ABSTRACT

Let $H:\mathbb{R}^3\to\mathbb{R}$ be a C^1 mapping such that $H(p)\to H_\infty>0$ as $|\,p\,|\to\infty$. We show that when H satisfies some global conditions then there exists an H-bubble, namely a sphere S in $\mathbb{R}^3$ such that the mean curvature of S at any regular point $p\in S$ equals $H(p)$.

1. Introduction

In this paper, we make a contribution to the following problem, raised by Yau in [1]: *"Let H be a real-valued function on* $\mathbb{R}^3$*. Find (reasonable) conditions on H to ensure that one can find a closed surface with prescribed genus in* $\mathbb{R}^3$ *whose mean curvature is given by H"*. In particular, we are interested in the existence of $\mathbb{S}^2$-type surfaces in $\mathbb{R}^3$ with prescribed mean curvature H.

Spheres in $\mathbb{R}^3$ with mean curvature H can be characterized as parametric surfaces, or more precisely as nonconstant solutions of the problem

$$\begin{cases}\Delta u = 2H(u)u_x\wedge u_y & \text{on } \mathbb{R}^2\\ \displaystyle\int_{\mathbb{R}^2}|\nabla u|^2<\infty.\end{cases}\tag{1.1}$$

If H satisfies suitable smoothness and growth assumptions (see [2–5]), then any weak solution to (1.1) is actually a classical solution. Moreover, it is conformal, and it parameterizes a closed surface S with area $\frac{1}{2}\int_{\mathbb{R}^2}|\nabla u|^2$. Furthermore, the surface S has mean curvature $H(p)$ at any point $p\in S$, with the exception of a finite number of singular points.

Problem (1.1) has also some relevance with regard to the Plateau problem for disc-type surfaces with prescribed mean curvature. We cite for instance [6–8] for a discussion of this feature.

Following [9], we will call an H-*bubble* any nonconstant solution of (1.1). We are interested in finding sufficient global conditions for the existence of H-bubbles when H approaches a positive constant at infinity, that is,

$$H(p)\to H_\infty \quad \text{as } |p|\to\infty,\ \text{ for some } H_\infty\in(0,\infty).$$

If $H\equiv H_\infty\in(0,\infty)$ is constant, then spheres of radius H_∞^{-1} centered at any point in $\mathbb{R}^3$ are the only H-bubbles (i.e., they admit parameterizations solving (1.1); see Lemma 0.1 in [10]).

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