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Clarke coderivatives of efficient point multifunctions in parametric vector optimization[☆]

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ABSTRACT

The paper is devoted to the study of the Clarke/circatangent coderivatives of the efficient point multifunction of parametric vector optimization problems in Banach spaces. We provide inner/outer estimates for evaluating the Clarke/circatangent coderivative of this multifunction in a broad class of conventional vector optimization problems in the presence of geometrical, operator and (finite and infinite) functional constraints. Examples are given for analyzing and illustrating the obtained results.

1. Introduction

This paper is motivated by sensitivity analysis in parametric vector optimization problems. We first give some notation and definitions.

Let $f : P \times X \to Y$ be a vector function, $C : P \rightrightarrows X$ be a multifunction where P, X and Y are Banach spaces. Given a pointed (i.e., $K \cap (-K) = \{0\}$) closed convex cone $K \subset Y$, we consider the following *parametric vector optimization problem*

$$\min_K \{f(p, x) \mid x \in C(p)\} \tag{1.1}$$

depending on the *parameter* $p \in P$. Here, x is a *decision variable* and the cone K induces a partial order $\preceq_K$ on Y, i.e.,

$$y \preceq_K y' \Leftrightarrow y' - y \in K, \quad y, y' \in Y. \tag{1.2}$$

The "$\min_K$" in (1.1) is understood with respect to the ordering relation $\preceq_K$ from (1.2).

We say that $y \in A$ is an *efficient point* of a subset $A \subset Y$ with respect to K and write $y \in \operatorname{Min} A$, if and only if $(y - K) \cap A = \{y\}$. If $A = \emptyset$, then we stipulate that $\operatorname{Min} A = \emptyset$.

Let $F : P \rightrightarrows Y$ be a multifunction given by

$$F(p) = (f \circ C)(p) := f(p, C(p)) = \{f(p, x) \mid x \in C(p)\}. \tag{1.3}$$

We put

$$\mathcal{F}(p) = \operatorname{Min} F(p), \quad p \in P \tag{1.4}$$

and call $\mathcal{F} : P \rightrightarrows Y$ the *efficient point multifunction* of (1.1).

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