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Optimal control for multi-phase fluid Stokes problems

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ABSTRACT

Optimal control for a system consistent of the viscosity dependent Stokes equations coupled with a transport equation for the viscosity is studied. Motivated by a lack of sufficient regularity of the adjoint equations, artificial diffusion is introduced to the transport equation. The asymptotic behavior of the regularized system is investigated. Optimality conditions for the regularized optimal control problems are obtained and again the asymptotic behavior is analyzed. The lack of uniqueness of solutions to the underlying system is another source of difficulties for the problem under investigation.

1. Introduction

The focus of this work is to establish an approach for optimal control multi-phase fluid flow. More specifically we consider the problem

$$\min J(\eta, \mathbf{u}) = \frac{1}{2}\|\eta - \widetilde{\eta}\|^2_{L^2(Q)} + \frac{\alpha}{2}\|\mathcal{B}\mathbf{u}\|^2_{L^2(Q)}, \tag{1.1}$$

subject to

$$\begin{cases} \mathbf{y}_t - \operatorname{div}\,(\eta(\nabla \mathbf{y})) + \nabla p = \mathcal{B}\mathbf{u}, \\ \operatorname{div}\mathbf{y} = 0, \qquad \mathbf{y}|_{\partial\Omega} = \mathbf{0}, \qquad \mathbf{y}|_{t=0} = \mathbf{y}_0, \\ \eta_t + \mathbf{y}\cdot\nabla\eta = 0, \\ \eta|_{t=0} = \eta_0. \end{cases} \tag{1.2}$$

Let us describe the various terms in this problem formulation. Here Ω is a bounded domain in $\mathbb{R}^2$ with smooth boundary, time interval $T > 0$ is fixed and $Q = (0, T) \times \Omega$. The spatio-temperally dependent vector field $\mathbf{y}$ presents the velocity of the fluid, p its pressure, and η is the nonconstant viscosity of the fluid. Further $\mathbf{y}_0$ and η_0 are the initial velocity and viscosity respectively. The control variable is denoted by $\mathbf{u}$, it may act on the subset $\tilde{\Omega} \subset \Omega$. Control operator $\mathcal{B}$ is a bounded linear operator from $L^2(\mathbf{L}^2(\tilde{\Omega}))$ to $\mathbf{L}^2(Q)$, which will be defined in a later section. The control problem consists in finding $\mathbf{u}$ such that the corresponding state-control vector $(\mathbf{y}, \eta, p, \mathbf{u})$ minimizes $J(\eta, \mathbf{u})$, where $\tilde{\eta}$ is given and fixed.

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