Nonlinear Analysis 74 (2011) 6788–6804

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Hybrid pseudo-viscosity approximation schemes for systems of equilibrium problems and fixed point problems of infinite family and semigroup of non-expansive mappings

Hossein Piri

Department of Mathematics, University of Bonab, Bonab 55517-61167, Iran

ARTICLE INFO

Article history:
Received 8 September 2010
Accepted 10 June 2011
Communicated by Enzo Mitidieri

MSC:
47H09
47H10
47H20
43A07
47J25

Keywords:
Fixed point
Hybrid pseudo-viscosity approximation schemes
Non-expansive mapping
Amenable semigroup
Equilibrium problem

ABSTRACT

In this paper, we introduce hybrid pseudo-viscosity approximation schemes with strongly positive bounded linear operators for finding a common element of the set of solutions to a system of equilibrium problems, the set of fixed points of an infinite family and left amenable semigroup of non-expansive mappings in the frame work of Hilbert spaces. Our goal is to prove a result of strong convergence for hybrid pseudo-viscosity approximation schemes to approach a solution of systems of equilibrium problems which is also a common fixed point of an infinite family and left amenable semigroup of non-expansive mappings. The results presented in this paper can be treated as an extension and improvement of the corresponding results announced by Ceng et al. [L.C. Ceng, Q.H. Ansari, and J.C. Yao, Hybrid pseudo-viscosity approximation schemes for equilibrium problems and fixed point problems of infinitely many non-expansive mappings, Nonlinear Analysis 4 (2010) 743–754] and many others.

1. Introduction

Let H be a real Hilbert space with inner product $\langle ., .\rangle$ and norm $\|.\|$. Let C be a nonempty closed convex subset of H and F be a bi-function of $C \times C$ into $\mathbb{R}$, where $\mathbb{R}$ is the set of real numbers. The equilibrium problem for $F : C \times C \to \mathbb{R}$ is to find $x \in C$ such that

$$F(x, y) \geq 0, \quad \forall y \in C. \tag{1}$$

The set of solution of (1) is denoted by $\mathrm{EP}(F)$. It is a unified model of several problems, namely, variational inequality problems, complementarity problems, saddle point problems, optimization problems, and fixed point problems; see for example [1,2] and the references therein. A mapping T of C into itself is called non-expansive if $\|Tx - Ty\| \leq \|x - y\|$, for all $x, y \in C$. By $\mathrm{Fix}(T)$, we denote the set of fixed point of T i.e., $\mathrm{Fix}(T) = \{x \in H : Tx = x\}$. It is well known that $\mathrm{Fix}(T)$ is closed and convex. Assume A is strongly positive; that is, there is a constant $\overline{\gamma} > 0$ with the property

$$\langle Ax, x\rangle \geq \overline{\gamma}\|x\|^2, \quad \forall x \in H.$$

E-mail address: h.piri@bonabetu.ac.ir.

0362-546X/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2011.06.056