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Existence, nonexistence and asymptotic behavior of boundary blow-up solutions to $p(x)$-Laplacian problems with singular coefficient[☆]

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ABSTRACT

This paper investigates the problem

$$\begin{cases} -\Delta_{p(x)}u+\rho(x)f(x,u)=0 & \text{in } \Omega, \\ u(x)\to+\infty & \text{as } d(x,\partial\Omega)\to 0, \end{cases}$$

where $-\Delta_{p(x)}u=-div(|\nabla u|^{p(x)-2}\nabla u)$ is called the $p(x)$-Laplacian, and $\rho(x)$ is a singular coefficient. The existence and nonexistence of boundary blow-up solutions is discussed, and the asymptotic behavior of boundary blow-up solutions is given. In particular, we do not assume radial symmetric conditions, and the pointwise different exact blow-up rate of solutions has been discussed.

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1. Introduction

The study of differential equations and variational problems with variable exponent growth conditions is a new and interesting topic. It arises from nonlinear elasticity theory, electro-rheological fluids, image processing, etc. (see [1–4]). Many results have been obtained on this kind of problem, for example [1–20]. On the existence of solutions for $p(x)$-Laplacian problems, we refer to [6,9,13,14,19]. In this paper, we consider the following problem

$$\text{(P)}\quad \begin{cases} -\Delta_{p(x)}u+\rho(x)f(x,u)=0 & \text{in } \Omega, \\ u(x)\to+\infty & \text{as } d(x,\partial\Omega)\to 0, \end{cases}$$

where $-\Delta_{p(x)}u=-\mathrm{div}(|\nabla u|^{p(x)-2}\nabla u)$, $\Omega\subset\mathbb{R}^N$ $(N\geq 2)$ is a bounded domain with C^2 boundary $\partial\Omega$. Our aim is to discuss the existence, nonexistence and pointwise different exact blow-up rate of solutions for problem (P). The operator $-\Delta_{p(x)}u=-\mathrm{div}(|\nabla u|^{p(x)-2}\nabla u)$ is called $p(x)$-Laplacian. Especially, if $p(x)\equiv p$ (a constant), (P) is the well known p-Laplacian problem.

Throughout the paper, we assume that $p(x)$, $\rho(x)$ and $f(x,u)$ satisfy

(H$_1$) $p(\cdot)\in C^2(\overline{\Omega})$ and satisfies

$$1<p^-\leq p^+<N,\quad \text{where } p^-=\inf_{\Omega}p(x),\ p^+=\sup_{\Omega}p(x).$$

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