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Effective rates of convergence for Lipschitzian pseudocontractive mappings in general Banach spaces☆

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ABSTRACT

This paper gives an explicit and effective rate of convergence for an asymptotic regularity result $\|Tx_n - x_n\| \to 0$ due to Chidume and Zegeye in 2004 [14] where (x_n) is a certain perturbed Krasnoselski–Mann iteration schema for Lipschitz pseudocontractive self-mappings T of closed and convex subsets of a real Banach space. We also give a qualitative strengthening of the theorem by Chidume and Zegeye, by weakening the assumption of the existence of a fixed point. For the bounded case, our bound is polynomial in the data involved.

1. Introduction

A fundamental theorem in the early stages of metric fixed point theory is the following theorem by Krasnoselski which, apart from showing the existence of at least one fixed point, also provides a sequence approximating one of these fixed points.

Theorem (*Krasnoselski [1]*). *Let K be a nonempty closed, convex and bounded subset of a uniformly convex Banach space X and let T be a nonexpansive mapping of K into a compact subset of K. Then for every $x_0 \in K$, the sequence*

$$x_{k+1} := \frac{x_k + Tx_k}{2}$$

converges strongly to a fixed point $z \in K$ of T.

Finding a uniform rate of convergence for the Picard iteration of T (depending on the starting point x_0 and the mapping or, in fact, just a bound on its initial displacement $\|x_0 - T(x_0)\|$) of strict contractions is trivial as the Banach fixed point theorem already provides this. The second author has shown in [2] that there is no effective procedure to compute a rate of convergence uniformly dependent on the data x_0 and T for the above so-called Krasnoselski iteration where T is nonexpansive.

In [2] it is also shown that one can, however, extract from Krasnoselski's proof a uniform bound for the asymptotic regularity of x_k, *i.e.*, for $\|Tx_k - x_k\| \to 0$, which only depends on the modulus of (uniform) convexity and the diameter of the set K (without any compactness condition needed). This was further generalized in [3] to a quantitative version of a theorem due to Groetsch [4] (see also [5]) for the Krasnoselski–Mann iterations

$$x_{n+1} := (1 - c_n)x_n + c_n Tx_n$$

for (c_n) in [0, 1] with $\sum c_n(1 - c_n) = \infty$.

☆ The results of this paper are (in a somewhat improved form) from the Bachelor Thesis of the first author (Körnlein (2010) [28]) written under the supervision of the second author.

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