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The number of solutions for a class of nonlocal nonhomogeneous gradient operator equations[☆]

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ABSTRACT

This paper deals with the number of solutions for a class of nonlocal nonhomogeneous gradient operator equations of the form $a(I(u))I'(u) = f$, where $I \in C^1(X, \mathbb{R})$, X is a reflexive Banach space, $I(0) = 0$, I is even and strictly convex, $\frac{I(u)}{\|u\|} \to +\infty$ as $\|u\| \to \infty$, $I' : X \to X^*$ is a bounded homeomorphism but is not necessarily homogeneous, $a : (0, +\infty) \to \mathbb{R}$ is continuous, $f \in X^* \setminus \{0\}$. Some properties and examples of such a functional I are given. Some results on the number of solutions of the nonlocal equation are obtained.

1. Introduction

After the work of Kirchhoff [1], various nonlocal differential equations, also called Kirchhoff type equations, have been studied extensively and many interesting results have been obtained, for example, for the p-Kirchhoff equations (including the case of $p = 2$) see e.g. [2–19], for the $p(x)$-Kirchhoff equations see e.g. [20–23], and for the $\overrightarrow{p}(x)$-Kirchhoff equations see [24].

We are interested in the number of solutions of the nonlocal differential equation. We can see that, in this respect, there are some essential differences between the homogeneous operator case and the nonhomogeneous operator case. In order to explain our idea, let us compare the $p(x)$-Kirchhoff equation with the p-Kirchhoff equation. It is well known that the p-Laplacian is $(p-1)$-homogeneous but the $p(x)$-Laplacian, when $p(\cdot)$ is not a constant, is nonhomogeneous. Consider firstly the following p-Kirchhoff equation

$$\begin{cases} -a\left(\int_\Omega \frac{|\nabla u|^p}{p} \mathrm{d}x\right) \Delta_p u = f & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain, $p \in (1, +\infty)$, $-\Delta_p u = -\mathrm{div}\left(|\nabla u|^{p-2}\nabla u\right)$ is the p-Laplacian, $a : (0, +\infty) \to (0, +\infty)$ is continuous, and $f \in (W_0^{1,p}(\Omega))^* \setminus \{0\} = \left(\mathcal{D}_0^{1,p}(\Omega)\right)^* \setminus \{0\}$. In the case where $a(t) \equiv 1$, problem (1.1) becomes

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