Nonlinear Analysis 74 (2011) 2175–2190

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

On decay and blow-up of the solution for a viscoelastic wave equation with boundary damping and source terms

Wenjun Liu [a,*], Jun Yu [b]

[a] College of Mathematics and Physics, Nanjing University of Information Science and Technology, Nanjing 210044, China
[b] Department of Mathematics and Statistics, University of Vermont, Burlington, VT 05401, USA

ARTICLE INFO

Article history:
Received 19 September 2010
Accepted 10 November 2010

MSC:
35B40
35B35
35L70
93D20

Keywords:
Viscoelastic wave equation
Exponential decay
Blow-up
Boundary damping
Boundary source

ABSTRACT

In this paper we consider the decay and blow-up properties of a viscoelastic wave equation with boundary damping and source terms. We first extend the decay result (for the case of linear damping) obtained by Lu et al. (On a viscoelastic equation with nonlinear boundary damping and source terms: Global existence and decay of the solution, Nonlinear Analysis: Real World Applications 12 (1) (2011), 295–303) to the nonlinear damping case under weaker assumption on the relaxation function $g(t)$. Then, we give an exponential decay result without the relation between $g'(t)$ and $g(t)$ for the linear damping case, provided that $\|g\|_{L^1(0,\infty)}$ is small enough. Finally, we establish two blow-up results: one is for certain solutions with nonpositive initial energy as well as positive initial energy for both the linear and nonlinear damping cases, the other is for certain solutions with arbitrarily positive initial energy for the linear damping case.

1. Introduction

In this work, we investigate the following viscoelastic wave equation with boundary damping and source terms

$$\begin{cases} u_{tt} - \Delta u + \int_0^t g(t-\tau)\Delta u(x,\tau)\,d\tau = 0, & (x,t) \in \Omega \times (0,\infty), \\ u(x,t) = 0, & (x,t) \in \Gamma_0 \times [0,\infty), \\ \dfrac{\partial u}{\partial \nu} - \displaystyle\int_0^t g(t-\tau)\frac{\partial u}{\partial \nu}(\tau)\,d\tau + |u_t|^{m-2}u_t = |u|^{p-2}u, & (x,t) \in \Gamma_1 \times [0,\infty), \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), & x \in \Omega. \end{cases} \tag{1.1}$$

Here Ω is a bounded domain in $\mathbb{R}^n$ ($n \geq 1$) with a smooth boundary $\partial\Omega = \Gamma_0 \cup \Gamma_1$, $\Gamma_0 \cap \Gamma_1 = \emptyset$, where Γ_0 and Γ_1 are measurable over $\partial\Omega$, endowed with the $(n-1)$-dimensional Lebesgue measures $\lambda_{n-1}(\Gamma_i)$, $i = 0, 1$, ν is the unit outward normal to $\partial\Omega$, g is a positive function, and $m \geq 2$, $p > 2$.

Problem (1.1) arises in the theory of viscoelasticity and describes the spread of strain waves in a viscoelastic configuration (see [1–3]). The global existence of the solution and general decay of the energy (when $m = 2$) for this problem have been established by Lu et al. in [2] very recently. The aim of this paper is to further study the decay (when $m \geq 2$) and blow-up propositions of the solution for the same problem.

* Corresponding author.
E-mail addresses: wjliu@nuist.edu.cn, wjliu.cn@gmail.com (W. Liu).

0362-546X/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2010.11.022