Nonlinear Analysis 74 (2011) 5480–5494

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Sign-changing and multiple solutions of the Sturm–Liouville boundary value problem via invariant sets of descending flow☆

Yu Tian [a,*], Weigao Ge [b]

[a] School of Science, Beijing University of Posts and Telecommunications, 91 box, Beijing 100876, PR China

[b] Department of Applied Mathematics, Beijing Institute of Technology, Beijing 100081, PR China

ARTICLE INFO

Article history:
Received 8 November 2010
Accepted 11 May 2011
Communicated by Enzo Mitidieri

MSC:
34B15
35A15

Keywords:
Sign-changing solution
The Sturm–Liouville boundary value problem
Invariant set of descending flow
Critical point

ABSTRACT

In this paper, we prove the existence of sign-changing and multiple solutions for the second-order Sturm–Liouville boundary value problem

$$\begin{cases} -Lu = f(x,u), & x \in [0,1] \\ R_1(u) = 0, & R_2(u) = 0, \end{cases}$$

where $Lu = (p(x)u')' - q(x)u$ is the Sturm–Liouville operator, $R_1(u) = \alpha u'(0) - \beta u(0)$ and $R_2(u) = \gamma u'(1) + \sigma u(1)$. The technical approach is fully based on minimax methods and invariant sets of descending flow.

1. Introduction

It is interesting to studying the existence of sign-changing solutions. Invariant sets of descending flow defined by a pseudogradient vector field of a functional in a Banach space plays an important role in the existence of sign-changing solutions. The method was first proposed by Sun [1]. For the properties of invariant sets of descending flow and applications, please refer to [2–9].

The Dirichlet boundary value problem

$$\begin{cases} -\Delta u = f(x,u), & x \in \Omega, \\ u|_{\partial\Omega} = 0 \end{cases}$$

has been studied in [5,1] by invariant sets of descending flow. In [10,11]

$$\begin{cases} -\left(a + b\int_\Omega |\nabla u|^2\right)\Delta u = f(x,u), & \text{in } \Omega \\ u = 0 \quad \text{on } \partial\Omega \end{cases}$$

was studied. Sign-changing solutions have been obtained by invariant sets of descending flow.

☆ Project 11001028 supported by National Science Foundation for Young Scholars, Project BUPT2009RC0704 supported by Chinese Universities Scientific Fund and Project 11071014 supported by National Science Foundation of PR China.

* Corresponding author.
E-mail address: tianyu2992@163.com (Y. Tian).

0362-546X/$ – see front matter
doi:10.1016/j.na.2011.05.035