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Global existence, uniform decay and blow-up of solutions for a system of Petrovsky equations

Gang Li, Yanan Sun, Wenjun Liu *

College of Mathematics and Physics, Nanjing University of Information Science and Technology, Nanjing 210044, China

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ABSTRACT

In this paper we investigate global existence, uniform decay and blow-up of solutions for the following Petrovsky equations:

$$\begin{cases} u_{tt} + \Delta^2 u + |u_t|^{p-1} u_t = F_u(u, v), & (x, t) \in \Omega \times [0, T), \\ v_{tt} + \Delta^2 v + |v_t|^{q-1} v_t = F_v(u, v), & (x, t) \in \Omega \times [0, T), \end{cases}$$

where Ω is a bounded domain of R^n $(n = 1, 2, 3)$ having a smooth boundary and F is a C^1 function given by

$$F(u, v) = \alpha\, |u + v|^{r+1} + 2\beta\, |uv|^{\frac{r+1}{2}}, \quad r \geq 3, \alpha > 1, \beta > 0.$$

For the case of $p = q = 1$, we obtain the blow-up of solutions and the lifespan estimates for four different ranges of initial energy; for the case of $1 < p, q < r$, we show the blow-up of solutions when the initial energy is negative, or nonnegative at less than the mountain pass level value. Global existence of solutions is proved by the potential well theory, and decay estimates of the energy function are established by using Nakao's inequality.

1. Introduction

In this paper we consider the following initial–boundary value problem:

$$\begin{cases} u_{tt} + \Delta^2 u + |u_t|^{p-1} u_t = F_u(u, v), & (x, t) \in \Omega \times [0, T), \\ v_{tt} + \Delta^2 v + |v_t|^{q-1} v_t = F_v(u, v), & (x, t) \in \Omega \times [0, T), \\ u(x, 0) = u_0(x), \qquad u_t(x, 0) = u_1(x), & x \in \Omega, \\ v(x, 0) = v_0(x), \qquad v_t(x, 0) = v_1(x), & x \in \Omega, \\ u(x, t) = \partial_\nu u(x, t) = 0, \qquad v(x, t) = \partial_\nu v(x, t) = 0, & (x, t) \in \partial\Omega \times [0, T), \end{cases} \tag{1.1}$$

where $\Omega \subset R^n$ is a bounded domain with smooth boundary $\partial\Omega$ and so the divergence theorem can be applied, ν is the unit normal vector pointing toward the exterior of Ω, $\frac{\partial}{\partial \nu}$ denotes the normal derivative, $p, q \geq 1$, $T > 0$, and $F : R^2 \to R$ is a C^1 function given by

$$F(u, v) = \alpha\, |u + v|^{r+1} + 2\beta\, |uv|^{\frac{r+1}{2}},$$

where $r \geq 3$, $\alpha > 1$ and $\beta > 0$, which implies

$$F_u(u, v) = (r + 1) \left[\alpha\, |u + v|^{r-1} (u + v) + \beta\, |u|^{\frac{r-3}{2}} |v|^{\frac{r+1}{2}} u\right],$$

* Corresponding author.
E-mail addresses: wjliu@nuist.edu.cn, wjliu.cn@gmail.com (W. Liu).

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