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Multiple solutions of Sturm–Liouville boundary value problem via lower and upper solutions and variational methods☆

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ABSTRACT

In this paper, we prove the existence of multiple solutions for second order Sturm–Liouville boundary value problem

$$\begin{cases} -Lu = f(x, u), & x \in [0, 1] \\ R_1(u) = 0, & R_2(u) = 0, \end{cases}$$

where $Lu = (p(x)u')' - q(x)u$ is a Sturm–Liouville operator, $R_1(u) = \alpha u'(0) - \beta u(0)$, $R_2(u) = \gamma u'(1) + \sigma u(1)$. The technical approach is fully based on lower and upper solutions and variational methods. The interesting point is that the existence of four solutions and seven solutions is given.

1. Introduction

In recent years, there have been many papers studying the existence of solutions for boundary value problems, please refer to [1–8]. Agarwal et al. [9], Anuradha et al. [10], Erbe and Wong [11], Ge and Ren [12], Sun and Zhang [13], Zhang and Sun [14] have studied positive solutions of Sturm–Liouville boundary value problem by using fixed point theorem. Mao and Zhang [15] studied the existence of solutions for Kirchhoff type problems by using minimax methods and invariant sets of descending flow. Zhang and Perera [16] obtained the existence of positive, negative and sign-changing solutions of a class of nonlocal quasilinear elliptic boundary value problems using variational methods and invariant sets of descending flow. In papers [17,18], the existence of positive, negative and sign-changing solutions for asymptotically linear three-point boundary value problems was studied by using the topological degree theory and the fixed point index theory when the nonlinear term f is continuous and strictly increasing. In paper [39], Han and Li studied the existence of solutions for fourth order boundary value problem by using the critical point theory and the supersolution and subsolution method. Bonanno and Riccobono [4], Bonanno and Molica Bisci [19], Ricceri [20,21], Averna and Bonanno [22], Tian and Ge [23–26] studied positive solutions and multiple solutions for boundary value problems by using variational methods.

In papers [13,27], Sun and Zhang studied Sturm–Liouville boundary value problem

$$\begin{cases} -(L\varphi)(x) = h(x)f(\varphi(x)), & 0 < x < 1, \\ R_1(\varphi) = \alpha_1\varphi(0) + \beta_1\varphi'(0) = 0, \\ R_2(\varphi) = \alpha_2\varphi(1) + \beta_2\varphi'(1) = 0, \end{cases}$$

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