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Existence of quasi-periodic solutions and Littlewood's boundedness problem of sub-linear impact oscillators

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ABSTRACT

In this paper, it is shown by a series of transformations that how Moser's invariant curve theorem can be used to analyze the dynamical behavior of sub-linear Duffing-type equations with impact. We prove that all solutions are bounded, and that there are infinitely many periodic and quasi-periodic solutions in this impact case.

1. Introduction and main results

Impact oscillators of the form

$$\begin{cases} \ddot{x} + V'_x(x,t) = 0, & \text{for } x(t) > 0; \\ x(t) \geq 0; \\ \dot{x}(t_0^+) = -\dot{x}(t_0^-), & \text{if } x(t_0) = 0, \end{cases} \tag{1}$$

where $\dot{x}$ denotes dx/dt, serve as models of dynamical systems with discontinuity; cf. [1]. From the viewpoint of mechanics, these types of equations model the motion of a particle attached to a nonlinear spring and bouncing elastically against the fixed barrier ($x = 0$); cf. Fig. 1.

If $V'_x(0,t) \equiv 0$, Eq. (1) could always be simplified to a second order equation without impact by a centro-symmetric vector field. But many typical models could not be discussed as above. For example, the forced impact oscillators ($V'_x(0,t) = g(x) - p(t)$) do not satisfy that $V'_x(0,t) \equiv 0$.

Systems of the form (1) are special cases of vibro-impact systems; see e.g., [2]. They are also related to the Fermi accelerator, see e.g., [3], dual billiards, see e.g., [4], and certain models used in celestial mechanics, see e.g., [5]. The nonsmoothness caused by the impact limits the applications of many useful mathematical tools. However, there are still some interesting papers on the periodic, quasi-periodic and other regular motions for the impact oscillators; see [3,4,6–14] and the references therein.

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