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On cone metric spaces: A survey

Slobodanka Janković [a], Zoran Kadelburg [b], Stojan Radenović [c,*]

[a] Mathematical Institute SANU, Knez Mihailova 36, 11001 Beograd, Serbia

[b] University of Belgrade, Faculty of Mathematics, Studentski trg 16, 11000 Beograd, Serbia

[c] University of Belgrade, Faculty of Mechanical Engineering, Kraljice Marije 16, 11120 Beograd, Serbia

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ABSTRACT

Using an old M. Krein's result and a result concerning symmetric spaces from [S. Radenović, Z. Kadelburg, Quasi-contractions on symmetric and cone symmetric spaces, Banach J. Math. Anal. 5 (1) (2011), 38–50], we show in a very short way that all fixed point results in cone metric spaces obtained recently, in which the assumption that the underlying cone is normal and solid is present, can be reduced to the corresponding results in metric spaces. On the other hand, when we deal with non-normal solid cones, this is not possible. In the recent paper [M.A. Khamsi, Remarks on cone metric spaces and fixed point theorems of contractive mappings, Fixed Point Theory Appl. 2010, 7 pages, Article ID 315398, doi:10.1115/2010/315398] the author claims that most of the cone fixed point results are merely copies of the classical ones and that any extension of known fixed point results to cone metric spaces is redundant; also that underlying Banach space and the associated cone subset are not necessary. In fact, Khamsi's approach includes a small class of results and is very limited since it requires only normal cones, so that all results with non-normal cones (which are proper extensions of the corresponding results for metric spaces) cannot be dealt with by his approach.

1. Introduction

There are many generalizations of metric spaces: Menger spaces, fuzzy metric spaces, generalized metric spaces, abstract (cone) metric spaces or K-metric and K-normed spaces, rectangular metric and rectangular cone metric spaces . . .

In 2007, Huang and Zhang [1] introduced cone metric spaces, being unaware that they already existed under the name K-metric, and K-normed spaces that were introduced and used in the middle of the 20th century in [2–8]. In both cases the set $\mathbb{R}$ of real numbers was replaced by an ordered Banach space E. However, Huang and Zhang went further and defined the convergence via interior points of the cone by which the order in E is defined. This approach allows the investigation of cone spaces in the case when the cone is not necessarily normal. And yet, they continued with results concerned with the normal cones only. One of the main results from [1] is the following Banach Contraction Principle in the setting of normal cone spaces:

Theorem 1.1 ([1]). *Let (X, d) be a complete cone metric space over a normal solid cone. Suppose that a mapping $T : X \to X$ satisfies the contractive condition*

$$d(Tx, Ty) \preceq \lambda \cdot d(x, y) \tag{1.1}$$

* Corresponding author.
E-mail addresses: bobaj@mi.sanu.ac.rs (S. Janković), kadelbur@matf.bg.ac.rs (Z. Kadelburg), radens@beotel.net (S. Radenović).

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