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Global bifurcations of critical orbits of G-invariant strongly indefinite functionals

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ABSTRACT

Let $\mathbb{H}$ be a separable Hilbert space which is an orthogonal representation of a compact Lie group G and let $\Phi : \mathbb{H} \to \mathbb{R}$ be a G-invariant strongly indefinite functional of the class C^1. To study critical orbits of Φ we have defined the degree for G-invariant strongly indefinite functionals, which is an element of the Euler ring $U(G)$. Using this degree we have formulated the Rabinowitz alternative for G-invariant strongly indefinite functionals. The abstract results are applied to the study of global bifurcations of weak solutions of non-cooperative systems of elliptic differential equations.

1. Introduction

Throughout this article G stands for a compact Lie group, $(\mathbb{H}, \langle \cdot, \cdot \rangle)$ is a separable Hilbert G-representation and $C^k_G(\mathbb{H}, \mathbb{R})$ is the class of G-invariant functionals of the class C^k, $k \in \mathbb{N}$. Consider a functional $\Phi \in C^1_G(\mathbb{H}, \mathbb{R})$ of the form

$$\Phi(u) = \frac{1}{2}\langle Lu, u\rangle - \eta(u), \tag{1.1}$$

where $L : \mathbb{H} \to \mathbb{H}$ is a bounded, self-adjoint, linear, Fredholm operator and η is a nonlinear operator whose gradient $\nabla\eta \in C^0_G(\mathbb{H}, \mathbb{H})$ is completely continuous, where $C^k_G(\mathbb{H}, \mathbb{H})$ is the class of G-equivariant operators of the class C^k, $k \in \mathbb{N} \cup \{0\}$. We have a decomposition $\mathbb{H} = \mathbb{H}^- \oplus \mathbb{H}^0 \oplus \mathbb{H}^+$, where $\pm\langle Lu^\pm, u^\pm\rangle > 0$ for every $u^\pm \in \mathbb{H}^\pm$ and $\mathbb{H}^0 = \ker L$. Since L is a Fredholm operator, $\dim \mathbb{H}^0 < \infty$. If $\dim \mathbb{H}^- = \dim \mathbb{H}^+ = \infty$ then functional Φ is said to be strongly indefinite.

We study G-orbits of critical points of G-invariant strongly indefinite functionals of the form (1.1). We are mainly interested in the study of connected sets of critical orbits of such functionals. The study of critical points of strongly indefinite functionals is motivated by a number of problems from mathematical physics. They appear for example in the search for periodic solutions of Hamiltonian systems, wave equations and weak solutions of non-cooperative systems of elliptic differential equations.

Many topological and critical point theory tools have been developed for the study of critical points of strongly indefinite functionals. For instance the Morse theories have been defined in [1–9], for the Conley index theory see [10,11], the minimax methods have been considered in [12–15], and the spectral flow methods have been developed in [16–18]. In the presence of symmetries of a compact Lie group G critical orbits of G-invariant strongly indefinite functionals have been studied for instance in [19–22].

However, using these methods we cannot study connected sets of critical orbits of G-invariant functionals i.e. the continuation and global bifurcation of critical orbits; see [23–28] for discussion and examples.

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