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## Unilateral dynamic contact problem for viscoelastic Reissner–Mindlin plates

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#### ABSTRACT

The existence of solutions is proved for systems of dynamic Reissner–Mindlin equations expressing vibrations of viscoelastic plates. We consider the cases of short memory and singular memory material. Contact with the rigid support is considered.

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#### 1. Introduction and notation

In this paper we prove the existence of solutions to a unilateral dynamic contact problem for a viscoelastic Reissner–Mindlin plate.

In contrast to the classical Kirchhoff model with fourth-order plate equations for the unknown transversal deflection of the middle surface the Reissner–Mindlin model is more suitable for a thick plate, where the perpendicular filaments of the plate undergo a strain in deformation and hence the transverse shear deformation is taken into account. Originally it was created by Reissner [1,2] in order to receive three natural boundary conditions (for transverse shear force, bending and twisting moments). The dynamic model has been derived by Mindlin [3]. As in the deriving of the model the vector  $\nabla u$  of deflection derivatives is substituted by the vector  $\varphi$  of angles of rotations of the cross-sections  $x_1$  = constant and  $x_2$  = constant, the model contains a system of three second-order equations for the deflection and angles of rotation. The stability questions for the elastic and viscoelastic models are investigated in [4]. We concentrate here on the dynamic contact of viscoelastic Reissner–Mindlin plates with a rigid inner obstacle.

We were dealing with the analogous problems for elastic and viscoelastic von Kármán plates in [5–7]. Solvability of the dynamic domain contact problems for elastic structures in this case remains open. The case of viscoelastic membranes has been solved in [8]. We shall deal here with the plates made both of a short and of a singular memory viscoelastic material. The classical formulation of the problem is the initial-boundary value problem for a hyperbolic system containing the equation with a contact condition for the deflection of the middle surface of the plate and two equations for the unknown angles of rotation. We shall treat the cases of free and clamped plates. The existence of solutions is proved for appropriate approximate

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