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Nonlinear criteria for the existence of the exponential trichotomy in infinite dimensional spaces

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ABSTRACT

In this paper we obtain for the first time nonlinear conditions for the existence of the exponential trichotomy of skew-product flows in infinite dimensional spaces. We treat the most general case without any additional assumptions concerning the cocycle and without assuming a priori the existence of the projection families. We show that an inedit assembly of integral conditions imply the existence of the exponential trichotomy with all of its properties and we prove that the imposed conditions are also necessary. Our results generalize the previous studies on this topic and provide as particular cases many interesting situations, among which we mention the detection of the exponential trichotomy of general non-autonomous systems.

1. Introduction

Exponential trichotomy is the most complex asymptotic property of dynamical systems arising from the central manifold theory (see [1–15]). Starting from the idea that the center manifold of an equilibrium point of a dynamical system consists of orbits whose behavior around the equilibrium point is not controlled by either the attraction of the stable manifold or the repulsion of the unstable manifold, it was clear that the concept of exponential dichotomy describes a rather idealistic situation when the solution is either exponentially stable on the stable subspaces or exponentially unstable on the unstable subspaces (see [16,17,7,8,18]). Thus, with motivation from the properties arising in bifurcation theory (see [10], Chapter 6), it was the moment to introduce a new asymptotic concept called *exponential trichotomy* which reflects a deeper analysis of the behavior of solutions of dynamical systems. In this case the asymptotic behavior is described through the splitting of the main space into stable, unstable and central subspaces at each point from the flow's domain.

The concept of exponential trichotomy was introduced in the pioneering works of Elaydi and Hájek (see [6,7]), where the authors analyzed this property for the first time, both in the case of differential systems and in the case of nonlinear differential systems, pointing out significant properties of systems with exponential trichotomy. In the past few years, notable progress was made in the study of the exponential trichotomy of dynamical systems (see [1–5,8,9,11–14]) using diverse methods situated on the boundary between bifurcation theory and control, for various classes of trichotomy concepts. An important step was made by Zhu and Xu in [14], where they proved the Fredholm Alternative Lemma for a differential equation with exponential trichotomies, and, as a consequence, the main results could then be applied in order to obtain the persistence condition for heteroclinic orbits connecting non-hyperbolic equilibria. In [9] López-Fenner and Pinto proposed a nice study of discrete non-autonomous nonlinear systems possessing (h, k)-trichotomies. In [8]

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