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Center manifolds depending on a parameter☆

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ABSTRACT

For differential equations $u' = A(t)u + f(t, u, \lambda)$ obtained from sufficiently small C^1 perturbations of a nonuniform exponential trichotomy, we establish the C^1 dependence of the center manifolds on the parameter λ. Our proof uses the fiber contraction principle to establish the regularity property. We note that our argument also applies to linear perturbations, without further changes.

1. Introduction

We consider differential equations of the form

$$u' = A(t)u + f(t, u, \lambda), \tag{1}$$

with $f(t, 0, \lambda) = 0$ for every t and λ. Under certain "smallness" assumptions on the perturbation, when equation

$$u' = A(t)u \tag{2}$$

has some elliptic directions, one can establish the existence of center, stable, and unstable manifolds for each value of the parameter λ. In particular, when Eq. (2) has no unstable directions, all solutions converge exponentially to the center manifold. Therefore, the stability of the system is completely determined by the behavior on the center manifold. Thus, one often considers a reduction to the center manifold (see [1] for details and references). An exposition of the theory of center manifolds in the case of autonomous equations is given in [2], adapting results from [3]. See also [4,5] for the case of equations in infinite-dimensional spaces. We refer the reader to [6–8,2] for more details and further references.

Our main objective in this paper is to discuss the dependence of the center manifolds on the parameter λ. Particularly, in view of performing a reduction to the center manifolds, it is important to know not only whether the center manifolds are sufficiently regular for each λ, but also how they vary with λ. We established earlier in [9] that for each λ (with some "smallness" assumptions on the perturbation, see Section 2) there exists an invariant center manifold for the zero solution of Eq. (1). We emphasize that we consider the general context of *nonuniform* exponential behavior. In their "classical" formulation, center manifold theorems apply to flows for which Eq. (2) has a *uniform* exponential trichotomy. This means that the exponential estimates for the norms of its solutions are independent of the initial time. It turns out that the classical notion of (uniform) exponential behavior is very stringent for the dynamics and it is of interest to look for more general types

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