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Radially symmetric systems with a singularity and asymptotically linear growth

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ABSTRACT

We prove the existence of infinitely many periodic solutions for radially symmetric systems with a singularity of repulsive type. The nonlinearity is assumed to have a linear growth at infinity, being controlled by two constants which have a precise interpretation in terms of the Dancer–Fučik spectrum. Our result generalizes an existence theorem by Del Pino et al. (1992) [4], obtained in the case of a scalar second order differential equation.

1. Introduction

We consider a radially symmetric system in $\mathbb{R}^N$ of the type

$$\ddot{x} = -h(t, |x|)\frac{x}{|x|}, \tag{1}$$

where $h : \mathbb{R}\times]0, +\infty[\to \mathbb{R}$ is continuous, T-periodic with respect to its first variable, and has a singularity of repulsive type at the origin in its second variable. We look for non-collision periodic solutions, i.e., solutions which never attain the singularity.

As a first approach, one could look for periodic solutions whose orbits always stay on a given radius. Setting $\rho(t) = |x(t)|$, the study of these solutions involves the scalar equation

$$\ddot{\rho} + h(t, \rho) = 0, \tag{2}$$

which has been first considered by Lazer and Solimini in [1] and later investigated by many authors; see, e.g., [2–11]. Roughly speaking, the singularity being of repulsive type, one can expect that T-periodic solutions exist, provided that the force is attractive at some distance from the origin. However, some care must be taken in order to avoid what seems to be a kind of *resonance* at infinity. This fact is put in evidence in the following result by Del Pino, Manásevich and Montero.

Theorem 1 ([4]). *Let the following two assumptions hold.*

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