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# Metric subregularity for composite-convex generalized equations in Banach spaces☆

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ABSTRACT

In this paper we consider a convex-composite generalized constraint equation in Banach spaces. Using variational analysis technique, in terms of normal cones and coderivatives, we first establish sufficient conditions for such an equation to be metrically subregular. Under the Robinson qualification, we prove that these conditions are also necessary for the metric subregularity. In particular, some existing results on error bound and metric subregularity are extended to the composite-convexity case from the convexity case.



## 1. Introduction

Let $X$ be a Banach space and $\psi : X \to \mathbb{R} \cup \{+\infty\}$ be a proper lower semicontinuous function. Consider the following inequality

$$\psi(x) \leq 0. \tag{IE}$$

Let $S$ denote the solution set of (IE) and recall that (IE) has a local error bound at $a \in S$ if there exist $\eta, \delta \in (0, +\infty)$ such that

$$\eta d(x, S) \leq [\psi(x)]_+ \quad \text{for all } x \in B(a, \delta), \tag{1.1}$$

where $[\psi(x)]_+ := \max\{\psi(x), 0\}$ and $B(a, \delta)$ denotes the open ball with center $a$ and radius $\delta$. As observed in some papers (cf. [1,2]), the error bound property is identical to the weak sharp minimum property. It is known that the error bound and weak sharp minima have important applications in sensitivity analysis and convergence analysis of mathematical programming. Over the last two decades, the error bound and weak sharp minima have been extensively studied under various names (cf. [1,3–16] and the references therein). Let $F : X \rightrightarrows Y$ be a closed multifunction between Banach spaces $X$ and $Y$ and let $b \in Y$. The following generalized equation

$$b \in F(x) \tag{GE}$$

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