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Some fixed point theorems for s-convex subsets in p-normed spaces

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ABSTRACT

The existence of fixed points of operators on s-convex sets in p-normed spaces is studied, where $0 < p \leq 1, 0 < s \leq p$. By means of homeomorphisms the fixed point theorems of the types of Brouwer, Schauder, Kakutani, Rothe, Petryshyn, Altman, and Krasnosel'skii are proved for s-convex sets. As an application, the existence result of solutions for the game concerning s-convex–concave operators is given in product p-normed spaces.

1. Introduction and preliminaries

Let X be a linear space over K with the origin θ, where K is the field of real numbers or complex numbers. A p-norm on X is a nonnegative real-valued functional $\|\cdot\|_p$ on X with $0 < p \leq 1$, satisfying the following conditions:

(a) $\|x\|_p = 0$ if and only if $x = \theta$;
(b) $\|\lambda x\|_p = |\lambda|^p \|x\|_p$, for all $x \in X, \lambda \in K$;
(c) $\|x + y\|_p \leq \|x\|_p + \|y\|_p$, for all $x, y \in X$.

A linear space X on which there is a p-norm is called a p-normed space and is denoted by $(X, \|\cdot\|_p)$. If $p = 1$, then it is a usual normed space. A p-normed space is also a metric linear space with a translation invariant metric $d_p(x, y) = \|x - y\|_p$ for $x, y \in X$. It is a basic fact that the topology of every Hausdorff locally bounded topological linear space is given by some p-norm. The space $L^p(\mu)$ is a p-normed space based on the complete measure space $(\Omega, \mathcal{M}, \mu)$ with the p-norm given by

$$\|f(t)\|_p = \int_{\Omega} |f(t)|^p \mathrm{d}\mu, \quad \text{for } f \in L^p(\mu),$$

where Ω is a nonempty set, $\mathcal{M}$ is a σ-algebra in Ω, $\mu : \mathcal{M} \to [0, +\infty)$ is a positive measure, and

$$L^p(\mu) = \left\{ f \mid f : \Omega \to K \text{ is measurable, and } \int_{\Omega} |f(t)|^p \mathrm{d}\mu < +\infty \right\}.$$

If μ is the Lebesgue measure on $[0, 1]$, then it is customary to write $L^p[0, 1]$ instead of $L^p(\mu)$. If μ is the counting measure on $\Omega = \{1, \ldots, n\}$ or $\Omega = \{1, 2, \ldots\}$, then the corresponding spaces are denoted by $l^p(n)$ and l^p, respectively. $l^p(n)$ is

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