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Fractional Navier–Stokes equations and a Hölder-type inequality in a sum of singular spaces

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ABSTRACT

In this paper we study the local well-posedness of the fractional Navier–Stokes system with initial data belonging to a sum of two pseudomeasure-type spaces denoted by $PM^{a,b} := PM^a + PM^b$. The proof requires showing a Hölder-type inequality in $PM^{a,b}$, as well as establishing estimates of the semigroup generated by the fractional power of Laplacian $(-\Delta)^\gamma$ on these spaces.

1. Introduction and the statement of results

We study the Cauchy problem for the fractional Navier–Stokes equations in the whole space $\mathbb{R}^n$:

$$\begin{cases} u_t + (-\Delta)^\gamma u + (u\cdot\nabla)u + \nabla p = f, \quad x\in\mathbb{R}^n,\ t>0, \\ \operatorname{div} u = 0, \quad x\in\mathbb{R}^n,\ t>0, \\ u(x,t)\to 0 \quad \text{as } |x|\to\infty,\ t>0, \\ u(x,0) = u_0(x), \quad x\in\mathbb{R}^n, \end{cases} \tag{1.1}$$

where $n\geq 2$, $\gamma\in(\frac{1}{2},\frac{n+2}{4})$ and $(-\Delta)^\gamma$ is a fractional power of the Laplacian, which is defined through the Fourier transform as $((-\Delta)^\gamma\psi)\hat{}(\xi) = |\xi|^{2\gamma}\hat{\psi}(\xi)$.

Fluid mechanics models with fractional dissipation have been addressed in several recent works; see e.g. [1–5] and references therein. For $\gamma\geq\frac{n+2}{4}$, it is known that for each smooth initial value, system (1.1) possesses a global classical solution (see e.g. [4]). In the case $\gamma = 1$, (1.1) reduces to the well-known Navier–Stokes equations. In Clay Institute's list of prize problems, there is a famous open problem concerning whether smooth solutions for 3D Navier–Stokes equations blow up (or not) at a finite time T. In connection with this problem, a motivation arises for studying (in the range $\gamma<\frac{n+2}{4}$) system (1.1) in spaces that contain singular functions. In this direction, we are interested in the space $PM^{a,b} := PM^a + PM^b$,

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