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On scalar and vector ℓ-stable functions

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ABSTRACT

The notion of a scalar function that is ℓ-stable at a point is introduced in [D. Bednařík, K. Pastor, On second-order conditions in unconstrained optimization, Math. Program. Ser. A 113 (2008) 283–298]. In the present paper a characterization of the ℓ-stable functions is obtained. Further, the notion of an ℓ-stable function is generalized from scalar to vector functions. In an application, optimality conditions for constrained vector problems with ℓ-stable data are established.

1. Introduction

The notion of a scalar function that is ℓ-stable at a point is introduced by Bednařík and Pastor [1]. The motivation for this can be summarized as follows. Starting with the paper of Hiriart-Urruty et al. [2], second-order conditions for a point x^0 to be a solution for scalar or vector optimization problems with $C^{1,1}$ data near the reference point x^0 have been investigated. For scalar problems, results of this type can be seen in [3–6], and for vector problems, in [7–11]. Recall that a function f is called $C^{1,1}$ near x^0 if there is a neighborhood U of x^0 where f is Fréchet differentiable with a Lipschitz derivative. Observing that the aforementioned optimality conditions depend on properties only at the point x^0, one may think that the hypothesis of being $C^{1,1}$ near x^0 could be weakened in an appropriate way, such that these conditions remain true for problems with data satisfying the weakened hypothesis. The ℓ-stable function seems to be the appropriate relaxation. In [1] Bednařík and Pastor relax, for unconstrained scalar problems, the optimality condition presented in Ginchev et al. [12] from the functions being $C^{1,1}$ to them being ℓ-stable. The result obtained generalizes also those of Ben-Tal and Zowe [13], Cominetti and Correa [14] and Bednařík and Pastor [15]. Further, one reads in [1]: "Since $C^{1,1}$ functions appear in e.g. the augmented Lagrange method, the penalty function method and the proximal point method, and since the property to be $C^{1,1}$ function near some point requires stronger assumptions than the property to be ℓ-stable function at considered point, it seems to be useful to study the class of ℓ-stable functions". The eventual importance of the functions that are ℓ-stable at a point justifies their further study undertaken by Bednařík and Pastor in [16–18].

In the present paper we obtain an alternative characterization of a function that is ℓ-stable at a point and on this basis show the equivalence of the ℓ-stable and u-stable functions (the latter are defined as the ℓ-stable functions on replacing the lower derivative with the upper one). Further, we introduce the notion of an ℓ-stable vector function, develop the aforementioned characterization to vector functions, and prove optimality conditions for constrained vector optimization problems

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