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Solutions to a dyadic recurrent system and a certain action on $\mathcal{B}(\mathcal{H})$ induced by shifts[☆]

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ABSTRACT

Let $l^2(\mathbb{Z})$ be the Hilbert space of square summable double sequences of complex numbers with standard basis $\{e_n : n \in \mathbb{Z}\}$, and let us consider a bounded matrix A on $l^2(\mathbb{Z})$ satisfying the following system of equations

1. $\langle Ae_{2j}, e_{2i}\rangle = p_{ij} + a\langle Ae_j, e_i\rangle$;
2. $\langle Ae_{2j}, e_{2i-1}\rangle = q_{ij} + b\langle Ae_j, e_i\rangle$;
3. $\langle Ae_{2j-1}, e_{2i}\rangle = v_{ij} + c\langle Ae_j, e_i\rangle$;
4. $\langle Ae_{2j-1}, e_{2i-1}\rangle = w_{ij} + d\langle Ae_j, e_i\rangle$

for all i, j, where $P = (p_{ij}), Q = (q_{ij}), V = (v_{ij}), W = (w_{ij})$ are bounded matrices on $l^2(\mathbb{Z})$ and $a, b, c, d \in \mathbb{C}$.

It is clear that the solutions of the above system of equations introduces a new class of infinite matrices whose entries are related “dyadically”. In this paper, we will show that while the task of constructing these matrices explicitly using purely algebraic methods may appear to be very complicated and tedious, if not impossible, it can be carried out alternatively in a systematical and relatively simple way by applying the theory of Hardy classes of operators through a certain action on $\mathcal{B}(\mathcal{H})$ (space of bounded operators on $\mathcal{H}$) induced by a shift.

1. Motivations

Let $l^2(\mathbb{Z})$ be the Hilbert space of square summable double sequences of complex numbers with standard basis $\mathcal{E} = \{e_n : n \in \mathbb{Z}\}$. Also, let $P = (p_{ij}), Q = (q_{ij}), V = (v_{ij}), W = (w_{ij})$ be bounded matrices on $l^2(\mathbb{Z})$ and $a, b, c, d \in \mathbb{C}$. The purpose of this article is to study bounded solutions of the following system of equations

$$\begin{cases} \langle Ae_{2j}, e_{2i}\rangle = p_{ij} + a\langle Ae_j, e_i\rangle \\ \langle Ae_{2j}, e_{2i-1}\rangle = q_{ij} + b\langle Ae_j, e_i\rangle \\ \langle Ae_{2j-1}, e_{2i}\rangle = v_{ij} + c\langle Ae_j, e_i\rangle \\ \langle Ae_{2j-1}, e_{2i-1}\rangle = w_{ij} + d\langle Ae_j, e_i\rangle \end{cases} \tag{$\star$}$$

for all i, j (where $\langle\cdot,\cdot\rangle$ is the inner product). It is clear that if A is a solution of $(\star)$, then A can be determined by the entries $\{a_{00}, a_{01}, a_{10}, a_{11}\}$ (see Fig. 1).

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